NPS-MA-94-008

NAVAL POSTGRADUATE SCHOOL
Monterey, California

STRESSES IN SHIP PLATING

by

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Technical Report for Period
October 1993 - December 1994

Prepared for: Naval Postgraduate School
Monterey, CA 93943

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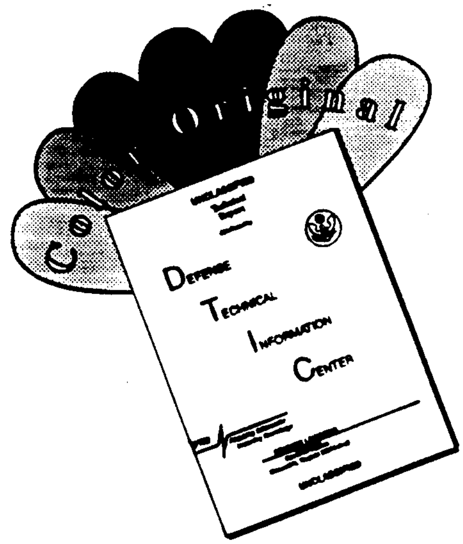

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This report was prepared in conjunction with research conducted for the Naval Postgraduate School and funded by the Naval Surface Warfare Center (Carderock Division) and by the Naval Postgraduate School.

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Accesion For	
NTIS CRA&I	X
DTIC TAB	☐
Unannounced	☐
Justification	
By	
Distribution /	
Availability Codes	
Dist	Avail and / or Special
A-1	

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION
Unclassified

1b. RESTRICTIVE MARKINGS

2a. SECURITY CLASSIFICATION AUTHORITY

2b. DECLASSIFICATION / DOWNGRADING SCHEDULE

3. DISTRIBUTION / AVAILABILITY OF REPORT
Approved for public release: distribution unlimited

4. PERFORMING ORGANIZATION REPORT NUMBER(S)
NPS-MA-94-008

5. MONITORING ORGANIZATION REPORT NUMBER(S)
NPS-MA-94-008

6a. NAME OF PERFORMING ORGANIZATION
Naval Postgraduate School

6b. OFFICE SYMBOL (If applicable)
MA

7a. NAME OF MONITORING ORGANIZATION
Naval Surface Warfare Center (Carderock Div.)
Naval Postgraduate School Research Program

6c. ADDRESS (City, State, and ZIP Code)
Monterey, CA 93943

7b. ADDRESS (City, State, and ZIP Code)
Naval Surface Warfare Center
Carderock Division
Bethesda, MD 20084-5000

8a. NAME OF FUNDING / SPONSORING ORGANIZATION
Naval Postgraduate School

8b. OFFICE SYMBOL (If applicable)
MA

9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
OM,N

8c. ADDRESS (City, State, and ZIP Code)
Monterey, CA 93943

10. SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO.	PROJECT NO	TASK NO	WORK UNIT ACCESSION NO.

11. TITLE (Include Security Classification)
Stresses in Ship Plating

12. PERSONAL AUTHOR(S)
D. A. Danielson, C. R. Steele, F. Fakhroo, A. S. Cricelli

13a. TYPE OF REPORT
Technical Report

13b. TIME COVERED
FROM 10/93 TO 12/94

14. DATE OF REPORT (Year, Month, Day)
December, 1994

15. PAGE COUNT
69

16. SUPPLEMENTARY NOTATION

17. COSATI CODES

FIELD	GROUP	SUB-GROUP

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
Stresses in Ship Plating

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

The subject of this paper is the mechanical behavior of rectangular plates subjected to a combination of axial compression and lateral pressure. Displacements and stresses are obtained from a Fortran code based on the von Kárman plate equations. The effects of various boundary conditions, nonlinearities, and imperfections are included.

20. DISTRIBUTION / AVAILABILITY OF ABSTRACT
☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS

21. ABSTRACT SECURITY CLASSIFICATION
Unclassified

22a. NAME OF RESPONSIBLE INDIVIDUAL
Donald Danielson

22b. TELEPHONE (Include Area Code)
408-656-2622

22c. OFFICE SYMBOL
MA/Dd

DD Form 1473, JUN 86

Previous editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE
UNCLASSIFIED

S/N 0102-LF-014-6603

STRESSES IN SHIP PLATING

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ABSTRACT

The subject of this paper is the mechanical behavior of rectangular plates subjected to a combination of axial compression and lateral pressure. Displacements and stresses are obtained from a Fortran code based on the von Kármán plate equations. The effects of various boundary conditions, nonlinearities, and imperfections are included.

INTRODUCTION

Fatigue life predictions require knowledge of the stresses in a ship under its operating conditions [Sikora, Dinsenbacher, and Beach (1983)]. Stiffened plates are a basic structural component of ships and submarines. The mathematical equations governing the deformation of thin elastic plates and methods for solution of these equations are well-known [Timoshenko and Woinowsky-Krieger (1959), Szilard (1974), and Hughes (1983)]. The objective of our work is to use these known mathematical methods to predict the stresses in typical ship plating.

Plates are subjected to axial tension and compression from longitudinal bending of a ship due to wave loads. In addition, plates on the bottom are subjected to lateral water pressure. Deck plating boundary conditions may be taken to be simply-supported , whereas bottom plating boundary conditions are more closely approximated as clamped. Deck plating may be considered to have an initial geometric imperfection, whereas bottom plating is bowed in by the water pressure.

We consider a rectangular plate with length a, width b, and thickness t. The structure is subjected to axial force F, and possibly uniform lateral pressure p (force per unit lateral area of the plate).

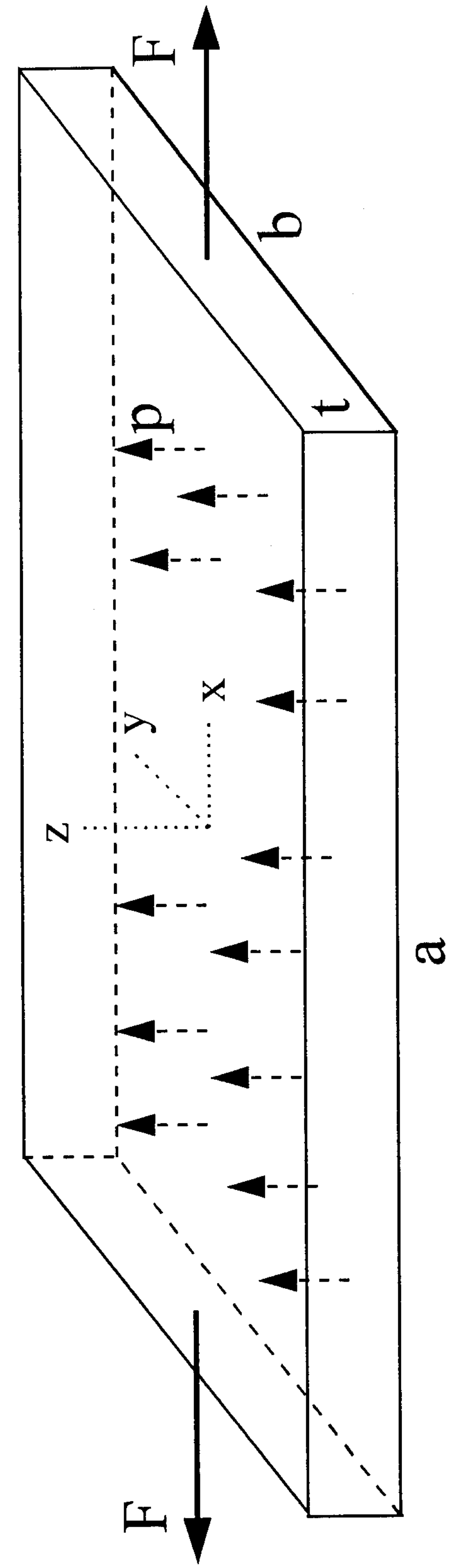
F
b
t
p
x
y
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PLATE EQUATIONS

The basic differential equations of nonlinear shallow shell theory are

$$D \nabla^4 w = p + \Phi_{,yy} (w_0 + w)_{,xx} + \Phi_{,xx} (w_0 + w)_{,yy} - 2\, \Phi_{,xy} (w_0 + w)_{,xy} \tag{1}$$

$$\frac{1}{Et} \nabla^4 \Phi = [(w_0 + w)_{,xy}^2 - (w_0 + w)_{,xx} (w_0 + w)_{,yy}] - (w_{0,xy}^2 - w_{0,xx}\, w_{0,yy}) \tag{2}$$

Here (x, y, z) are Cartesian coordinates measured from the center of the plate. $D = Et^3/12(1 - \nu^2)$ is the bending stiffness, E is Young's modulus, and ν is Poisson's ratio. $w_0(x, y)$ is the initial geometric imperfection of the plate midsurface in the z-direction, while $w(x, y)$ is the normal displacement. $\Phi(x, y)$ is the Airy stress function. Commas denote partial differentiation with respect to x or y; e.g,

$$\nabla^4 w = w_{,xxxx} + 2w_{,xxyy} + w_{,yyyy} = \frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^2 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}$$

We let $u(x, y)$ and $v(x, y)$ denote the displacements of the plate midsurface in the x and y directions, respectively. These in-plane displacements can be related to w and Φ by using the strain-displacement relations

$$\begin{aligned}
\epsilon_x &= u_{,x} + w_{0,x}\, w_{,x} + \tfrac{1}{2} w_{,x}^2 \\
\epsilon_y &= v_{,y} + w_{0,y}\, w_{,y} + \tfrac{1}{2} w_{,y}^2 \\
\epsilon_{xy} &= u_{,y} + v_{,x} + w_{0,x}\, w_{,y} + w_{0,y}\, w_{,x} + w_{,x}\, w_{,y}
\end{aligned}$$

and the constitutive relations

$$\begin{aligned}
N_x &= \frac{Et}{1 - \nu^2}(\epsilon_x + \nu \epsilon_y) \\
N_y &= \frac{Et}{1 - \nu^2}(\epsilon_y + \nu \epsilon_x) \\
N_{xy} &= \frac{Et}{2(1 + \nu)} \epsilon_{xy}
\end{aligned}$$

The membrane stress resultants (forces per unit length of side) are related to the stress function by

$$N_x = \Phi_{,yy}\,, \quad N_y = \Phi_{,xx}\,, \quad N_{xy} = -\Phi_{,xy}$$

The bending stress resultants (moments per unit length of side) are related to the normal displacement by

$$M_x = -D(w_{,xx} + \nu w_{,yy}), \quad M_y = -D(w_{,yy} + \nu w_{,xx}), \quad M_{xy} = D(1-\nu)w_{,xy} \tag{3}$$

The bending stresses (forces per unit area of cross section) acting on the top of sections parallel to the yz or xz planes, respectively are

$$\sigma_x = \frac{6M_x}{t^2}, \; \sigma_y = \frac{6M_y}{t^2}$$

We consider three different possible sets of boundary conditions:

Loosely-clamped

$$\begin{aligned} \text{On} \quad x &= \pm\tfrac{a}{2}: \quad w = w_{,x} = N_{xy} = 0, \quad u = \text{ constant}, \; \textstyle\int_{-\frac{b}{2}}^{\frac{b}{2}} N_x(\pm\frac{a}{2}, y)dy = F \\ \text{On} \quad y &= \pm\tfrac{b}{2}: \quad w = w_{,y} = N_{xy} = 0, \quad v = \text{constant}, \; \textstyle\int_{-\frac{a}{2}}^{\frac{a}{2}} N_y(x, \pm\frac{b}{2},)dx = 0 \end{aligned} \tag{4}$$

Rigidly-clamped

$$\begin{aligned} \text{On} \quad x &= \pm\tfrac{a}{2}: w = w_{,x} = N_{xy} = 0, \quad u = \text{ constant}, \textstyle\int_{-\frac{b}{2}}^{\frac{b}{2}} N_x(\pm\frac{a}{2}, y)dy = F \\ \text{On} \quad y &= \pm\tfrac{b}{2}: w = w_{,y} = N_{xy} = v = 0 \end{aligned} \tag{5}$$

Simply-supported

$$\begin{aligned} \text{On} \quad x &= \pm\tfrac{a}{2}: \quad w = w_{,xx} = N_{xy} = 0, \quad u = \text{ constant}, \; \textstyle\int_{-\frac{b}{2}}^{\frac{b}{2}} N_x \; (\pm\frac{a}{2}, y)dy = F \\ \text{On} \quad y &= \pm\tfrac{b}{2}: \quad w = w_{,yy} = N_{xy} = 0, \quad v = \text{ constant}, \; \textstyle\int_{-\frac{a}{2}}^{\frac{a}{2}} N_y(x, \pm\frac{b}{2})dx = 0 \end{aligned} \tag{6}$$

The classical buckling load of a simply-supported plate with an integral aspect ratio $\frac{a}{b} = 1, 2, 3, ...$ is given by $-F_{cr}$, where

$$F_{cr} = \frac{4\pi^2 D}{b}$$

SIMPLIFIED EQUATIONS

For normal displacements w which are smaller than the plate thickness t, the nonlinear terms in (1)-(2) are negligible and we can replace (1)-(2) with the linear approximation

$$N_x = \frac{F}{b}, \;\; N_y = constant\, C, \;\; N_{xy} = 0 \tag{7}$$

$$D \nabla^4 w = p + N_x(w_0 + w)_{,xx} + N_y(w_0 + w)_{,yy} \tag{8}$$

For the boundary conditions (4) and (6) where the lateral edges $y = \pm\frac{b}{2}$ are free to expand or contract $N_y = 0$, whereas for the boundary conditions (5) where the lateral edges $y = \pm\frac{b}{2}$ are restrained $N_y = \frac{\nu F}{b}$. If $p \neq 0$, equation (8) may be written in the nondimensionalized form

$$\overline{\nabla}^4 \overline{w} = 12(1-\nu^2) + \frac{4\pi^2 F}{F_{cr}}(\overline{w}_0 + \overline{w})_{,\overline{x}\overline{x}} + \frac{b^2 N_y}{D}(\overline{w}_0 + \overline{w})_{,\overline{y}\overline{y}}$$

where $\overline{w} = \frac{wEt^3}{pb^4}$, $\quad \overline{x} = \frac{x}{b}$, etc. The nondimensionalized bending stresses are then given by

$$\frac{\sigma_x t^2}{pb^2} = \frac{6M_x}{pb^2} = -\frac{1}{2(1-\nu^2)}(\overline{w}_{,\overline{x}\overline{x}} + \nu \overline{w}_{,\overline{y}\overline{y}}), \quad \text{etc.}$$

Note that in the case $w_0 = 0$ the normal displacement w increases linearly with the pressure p, whereas in the case $p = 0$ the normal displacement w increases linearly with the imperfection amplitude.

We can investigate the importance of nonlinearities by substituting the linear solution w to (8) into the right side of (2). The additional membrane stresses S_x, S_y, S_{xy} are then determined by

$$N_x = \frac{F}{b} + tS_x = \frac{F}{b} + \Phi_{,yy}\,, \;\; N_y = tS_y + C = \Phi_{,xx} + C, \;\; N_{xy} = tS_{xy} = -\Phi_{,xy}$$

$$\frac{1}{Et}\nabla^4\Phi = [(w_0+w),_{xy}^2 - (w_0+w),_{xx}(w_0+w),_{yy}] - (w_0,_{xy}^2 - w_0,_{xx}w_0,_{yy}) \tag{9}$$

If $p \neq 0$, Equation (9) may be written in the nondimensionalized form

$$\overline{\nabla}^4\overline{\Phi} = [(\overline{w}_0+\overline{w}),_{\overline{x}\overline{y}}^2 - (\overline{w}_0+\overline{w}),_{\overline{x}\overline{x}}(\overline{w}_0+\overline{w}),_{\overline{y}\overline{y}}] - (\overline{w}_0,_{\overline{x}\overline{y}}^2 - \overline{w}_0,_{\overline{x}\overline{x}}\overline{w}_0,_{\overline{y}\overline{y}})$$

where $\overline{\Phi} = \dfrac{\Phi E t^5}{p^2 b^8}$ The nondimensionalized additional membrane stresses are then given by

$$\frac{S_x t^2}{pb^2} = \frac{pb^4}{Et^4}\overline{\Phi},_{\overline{y}\overline{y}} \quad \text{etc.} \tag{10}$$

Note that the total longitudinal stress at the top of a plate cross section is $\dfrac{F}{tb} + \sigma_x + S_x$, etc.

SOLUTION METHOD

These equations may be solved numerically by superimposing a Fourier series particular solution plus Levy-type homogeneous solutions with coefficients determined by the prescribed boundary conditions. This method of solution has been recently used by Kwok, Kang, and Steele (1991), Bird and Steele (1991, 1992), and Kang, Wu, and Steele (1993). A thoroughly documented Fortran code was written to implement this solution.

Normal Displacement and Bending Stresses

The governing equations for an isotropic plate under axial compression and lateral pressure load with no imperfection are from (7) and (8)

$N_x = \dfrac{F}{b}$, $N_y = 0$ or $\dfrac{\nu F}{b}$, $N_{xy} = 0$

$D\nabla^4 w = p + N_x w,_{xx} + N_y w,_{yy}$

with the clamped boundary conditions from (4) or (5)

On $x = \pm\frac{a}{2}$: $w = w,_x = 0$

On $y = \pm\frac{b}{2}$: $w = w,_y = 0$

The solution to the problem above is obtained in several stages:

1) First a solution w_1 for constant pressure load on a clamped plate strip with $N_x = N_y = 0$ is obtained. This particular solution that satisfies

$$\nabla^4 w = \frac{p}{D} \tag{11}$$

is of the following form

$$w_1 = A\left(1 - \left(\frac{2y}{b}\right)^2\right)^2 \tag{12}$$

The solution w_1 has zero rotation and displacement along $y = \pm\frac{b}{2}$. From (3) the bending moment M_y is given by

$$M_y = -Dw_{,yy} = -\frac{16AD}{b^2}\left(1 - 3\left(\frac{2y}{b}\right)^2\right) \quad \text{and} \quad M_x = \nu M_y$$

Coefficient A in (12) is determined by substituting (12) in (11): $A = \frac{p}{D}\,\frac{b^4}{384}$.

The Fourier cosine series for (12) is given by

$$w_1 = \sum_{n\,\text{odd}} W_n \cos\frac{n\pi y}{b}$$

where

$$W_n = \frac{4}{b}\int_0^{\frac{b}{2}} w_1 \cos\frac{n\pi y}{b}\,dy = \frac{pb^4}{3D}\left(\frac{-1}{(n\pi)^3} + \frac{12}{(n\pi)^5}\right)\sin\frac{n\pi}{2}.$$

The particular solution w_1 has zero rotation but nonzero displacement along $x = \pm\frac{a}{2}$

2) In order to make the normal displacement zero along $x = \pm\frac{a}{2}$, we need to compute a correction term that will achieve this goal. This new term w_2 which is the solution of a rectangular plate with inplane resultants N_x and N_y is computed in the following way: A solution of the form $w_2 = \sum_{n\,\text{odd}} X_n(x)\cos\frac{n\pi y}{b}$ is assumed, where X_n is a function of x only and is determined from the condition that w_2 satisfies the plate equation

$$D\,\nabla^4 w = N_x\, w_{,xx} + N_y\, w_{,yy} \tag{13}$$

with zero rotation and displacement equal to $-w_1$ along $x = \pm\frac{a}{2}$. From the condition that w_2 satisfies (13), we obtain the solution

$$X_n\ (x) = A_n\ \cosh \nu_1 x + B_n\ \sinh\ \nu_1 x + C_n\ \cosh\ \nu_2 x + D_n\ \sinh\ \nu_2 x,$$

where ν_1 and ν_2 are solutions of

$$D\left(\nu^2 - \left(\frac{n\pi}{b}\right)^2\right)^2 + N_y\left(\frac{n\pi}{b}\right)^2 - N_x\nu^2 = 0 \tag{14}$$

The constants B_n and D_n can be taken to be zero due to the symmetry about the y-axis, and the remaining constants A_n and B_n can be determined by imposing the boundary conditions along $x = \pm\frac{a}{2}$. Then $w_1 + w_2$ is a solution for a plate under lateral pressure p and in-plane resultants N_x and N_y with zero displacement along all the edges and zero rotation along $x = \pm\frac{a}{2}$, but nonzero rotation along $y = \pm\frac{b}{2}$.

3) In order to make the rotation zero along $y = \pm\frac{b}{2}$ we first compute the Fourier coefficients of rotation on these edges and then obtain a third solution $w_3 = w_{31} + w_{32}$ with moments distributed along the edges corresponding to the desired rotations. A solution w_{31} sinusoidal in the x-direction is:

$$w_{31} = \sum_{m\ \mathrm{odd}} C_{1m}\ \cos\ \frac{m\pi x}{a}\left(\frac{\cosh\ \mu_1\, y}{\cosh\ \mu_1 \frac{b}{2}} - \frac{\cosh\ \mu_2\, y}{\cosh\ \mu_2\ \frac{b}{2}}\right)$$

where the parameters μ_1 and μ_2 are the roots of the polynomial

$$D\left(\mu^2 - \left(\frac{m\pi}{a}\right)^2\right)^2 + N_x\left(\frac{m\pi}{a}\right)^2 - N_y\mu^2 = 0$$

This solution is obtained in a similar manner as for solution w_2, except that in this case w_{31} is assumed to be of the form $w_{31} = \sum_{m\ \mathrm{odd}} Y_m\,(y)\ \cos\ \frac{m\pi x}{a}$ and $Y_m(y)$ is determined by imposing the zero displacement boundary condition along $y = \pm\frac{b}{2}$. As defined above, w_{31}

has zero displacement on all the edges and zero moment M_y along $x = \pm\frac{a}{2}$. The rotation along the edge $y = \frac{b}{2}$ is given by

$$\begin{aligned} roty = -(w_{31,y})\big|_{y=\frac{b}{2}} &= \sum_{m\text{ odd}} C_{1m} \cos\frac{m\pi x}{a}\left(-\mu_1 \tanh \mu_1 \frac{b}{2} + \mu_2 \tanh \mu_2 \frac{b}{2}\right) \\ &= \sum_{m\text{ odd}} C_{1m} \cos\frac{m\pi x}{a} D_{11m} \end{aligned} \tag{15}$$

The rotation along the edge $x = \frac{a}{2}$ is

$$rotx = -(w_{31,x})|_{x=\frac{a}{2}} = \sum_{n\ \text{odd}} C_{1m}\frac{m\pi}{a}\sin\frac{m\pi}{2}\left(\frac{\cosh \mu_1 y}{\cosh \mu_1 \frac{b}{2}} - \frac{\cosh \mu_2 y}{\cosh \mu_2 \frac{b}{2}}\right)$$

The expression in the previous parentheses is an even function of y, so it can be expanded in a Fourier cosine series:

$$rotx = \sum_{m\text{ odd}} C_{1m}\frac{m\pi}{a}\sin\frac{m\pi}{2}\left(\sum_{n\ odd} D_{12nm}\cos\frac{n\pi y}{b}\right) \tag{16}$$

where

$$\begin{aligned} D_{12nm} &= \frac{4}{b}\int_0^{\frac{b}{2}}\left(\frac{\cosh \mu_1 y}{\cosh \mu_1 \frac{b}{2}} - \frac{\cosh \mu_2 y}{\cosh \mu_2 \frac{b}{2}}\right)\cos\frac{n\pi y}{b}dy \\ &= \frac{4n\pi}{b^2}\sin\frac{n\pi}{2}\frac{\mu_1^2-\mu_2^2}{\left(\mu_1^2+(\frac{n\pi}{b})^2\right)\left(\mu_2^2+(\frac{n\pi}{b})^2\right)} \end{aligned}$$

The moment at the edge $y = \frac{b}{2}$ is:

$$\begin{aligned} M_y = D(-w_{31,yy} - \nu w_{31,xx})|_{y=\frac{b}{2}} &= \sum_{m\text{ odd}} C_{1m} D\left(-\mu_1^2+\mu_2^2\right)\cos\frac{m\pi x}{a} \\ &= \sum_{m\text{ odd}} F_{1m}\cos\frac{m\pi x}{a} \end{aligned}$$

Therefore

$$C_{1m} = \frac{F_{1m}}{D(-\mu_1^2+\mu_2^2)} \tag{17}$$

The solution sinusoidal in the y-direction which has zero displacement on the edges and zero moment M_y along $y = \pm\frac{b}{2}$ is :

$$w_{32} = \sum_{n\,\text{odd}} C_{2n} \cos\frac{n\pi y}{b} \left(\frac{\cosh\,\nu_1 x}{\cosh\,\nu_1 \frac{a}{2}} - \frac{\cosh\,\nu_2 x}{\cosh \nu_2 \frac{a}{2}} \right)$$

where ν_1 and ν_2 are roots of (14).

Now the rotation at the edge $x = \dfrac{a}{2}$ is

$$\begin{aligned} rotx = -(w_{32,x})|_{x=\frac{a}{2}} &= \sum_{n\,\text{odd}} C_{2n}\ \cos\ \frac{n\pi y}{b}\ \left(-\nu_1\ \tanh\ \frac{\nu_1 a}{2}\ + \nu_2\ \tanh\ \frac{\nu_2 a}{2}\right) \\ &= \sum_{n\,\text{odd}} C_{2n}\ \cos\ \frac{n\pi y}{b}\ D_{22n} \end{aligned} \tag{18}$$

The rotation at the edge $y = \dfrac{b}{2}$ is given by:

$$roty = -(w_{32,y})\big|_{y=\frac{b}{2}} \quad = \quad \sum_{n\,\text{odd}} C_{2n}\frac{n\pi}{b}\sin\frac{n\pi}{2}\left(\frac{\cosh\,\nu_1\,x}{\cosh\,\nu_1\frac{a}{2}} - \frac{\cosh\,\nu_2\,x}{\cosh\,\nu_2\,\frac{a}{2}}\right)$$

$$= \sum_{n\,\text{odd}} C_{2n}\frac{n\pi}{b}\sin\frac{n\pi}{2}\left(\sum_{m\,\text{odd}} D_{21nm}\cos\frac{m\pi x}{a}\right) \tag{19}$$

where

$$D_{21nm} = \frac{4m\pi}{a^2}\sin\frac{m\pi}{2}\left(\frac{\nu_1^2 - \nu_2^2}{\left(\nu_1^2 + \left(\frac{m\pi}{2}\right)^2\right)\left(\nu_2^2 + \left(\frac{m\pi}{a}\right)^2\right)}\right)$$

The moment at the edge $x = \dfrac{a}{2}$ is

$$\begin{aligned} M_x = D\left(-w_{32},_{xx} - \nu w_{32},_{yy}\right)|_{x=\frac{a}{2}} &= \sum_{n\,\text{odd}} C_{2n} D\left(-\nu_1^2 + \nu_2^2\right)\cos\frac{n\pi y}{b} \\ &= \sum_{n\,\text{odd}} F_{2n}\cos\frac{n\pi y}{b} \end{aligned}$$

Therefore

$$C_{2n} = \frac{F_{2n}}{D(-\nu_1^2 + \nu_2^2)} \tag{20}$$

In terms of the *Fourier coefficients* of quantities $rotx$, and $roty$, M_x, and M_y, we have the following relationships: From equations (16), (17), (18) and (20) we have:

$$[rotx] = [DM12] * [M_y] + [DM22] * [M_x] \tag{21}$$

where

$$\begin{aligned}
(DM12)_{nm} &= D_{12nm} * \frac{m\pi}{a} * \sin\frac{m\pi}{2} * \frac{1}{D(\mu_2^2 - \mu_1^2)} \\
(DM22)_{nm} &= D_{22n} * \frac{1}{D(\nu_2^2 - \nu_1^2)} * (\text{Identity})_{nm}
\end{aligned}$$

Similarly from (15), (17), (19) and (20) we have:

$$[roty] = [DM11] * [M_y] + [DM21] * [M_x] \tag{22}$$

where

$$(DM21)_{nm} = D_{21nm} * \frac{n\pi}{b} * \sin\frac{n\pi}{2} * \frac{1}{D(\mu_2^2 - \mu_1^2)},$$

$$(DM11)_{nm} = D_{11m} * \frac{1}{D(\mu_2^2 - \mu_1^2)} * (\text{Identity})_{nm}$$

Equations (21)-(22) can be written in the matrix notation:

$$\begin{bmatrix} DM11 & DM12 \\ DM21 & DM22 \end{bmatrix} \begin{bmatrix} M_y \\ M_x \end{bmatrix} = \begin{bmatrix} roty \\ rotx \end{bmatrix}$$

or

$$[DM] * [M] = rot$$

Here $[DM]$ is the flexibility matrix and the inverse $[DM]^{-1}$ is the stiffness matrix. To take advantage of the diagonal matrices $DM11$ and $DM22$, we write the following relations:

$$\begin{aligned}
[M_x] &= [M]^{-1} * ([rotx] - [DM21] * [DM11]^{-1} * [roty]) \\
[M_y] &= [DM11]^{-1} * ([roty] - [DM12] * [M_x]) \\
[M] &= [DM22] - [DM21] * [DM11]^{-1} * [DM12]
\end{aligned} \tag{23}$$

After computing the vector solution to equations (23), we have Fourier coefficients of M_x and M_y and from (17) and (20) we can compute the Fourier coefficients of w_3, the displacement corresponding to this bending moment. Then $w_1 + w_2 - w_3$ is the solution which has zero rotation on all the edges as well as zero displacement.

Additional Membrane Stresses

The governing equation for the additional membrane stresses in a plate with no imperfection is from (9)

$$\frac{1}{Et}\nabla^4\Phi = w_{,xy}^2 - w_{,xx}w_{,yy} \tag{24}$$

with the boundary conditions

$$\begin{array}{llll} \text{On } x = \pm\frac{a}{2}: & \int_{-\frac{b}{2}}^{\frac{b}{2}} \Phi_{,yy}\left(\pm\frac{a}{2}, y\right) dy = 0, & \int_{-\frac{b}{2}}^{\frac{b}{2}} \Phi_{,xy}\left(\pm\frac{a}{2}, y\right) dy = 0, & u = \text{constant} \\ \text{On } y = \pm\frac{b}{2}: & \int_{-\frac{a}{2}}^{\frac{a}{2}} \Phi_{,xx}\left(x, \pm\frac{b}{2}\right) dx = 0, & \int_{-\frac{a}{2}}^{\frac{a}{2}} \Phi_{,xy}\left(x, \pm\frac{b}{2}\right) dx = 0, & v = \text{constant.} \end{array} \tag{25}$$

The basic idea behind the solution method for calculating the in-plane Airy stress function Φ is the use of double Fourier expansions for both Φ and the out of plane displacement w. In order to solve the governing equation (24), we need to calculate the Fourier coefficients of w from the solution in the previous subsection. The Fourier expansions for w and Φ have the following form:

$$\begin{aligned} w &= \sum_{i\text{ even}}\sum_{j\text{ even}} w_{ij}\cos\left(\frac{i\pi x}{a}\right)\cos\left(\frac{j\pi y}{b}\right) \\ \Phi &= \sum_{i\text{ even}}\sum_{j\text{ even}} \Phi_{ij}\cos\left(\frac{i\pi x}{a}\right)\cos\left(\frac{j\pi y}{b}\right) \end{aligned} \tag{26}$$

With these expansions the boundary conditions (25) for Φ will be satisfied. The derivatives of w are calculated by direct differentiation of (26), and the products of derivatives are evaluated at each grid point. Then the discrete double Fourier cosine transform is used to obtain the Fourier coefficients of the right-hand side of (24), which are stored in the Gaussian curvature matrix K. The Fourier coefficients of Φ then follow from

$$\Phi_{ij} = \frac{EtK_{ij}}{\left(\left(\frac{i\pi}{a}\right)^2 + \left(\frac{j\pi}{b}\right)^2\right)^2} \qquad \text{for } i = 2, 4, \ldots, \quad j = 2, 4, \ldots.$$

The in-plane displacements u and v are obtained from the strain-displacement relations. An appropriate plane stress solution is added to satisfy the boundary conditions on u and v.

RESULTS

Normal Displacement and Bending Stresses

We first solved the linearized equations (7)-(8) with no imperfection and the loosely-clamped boundary conditions (4). The shapes of the deflected midsurfaces and values of the nondimensionalized longitudinal and transverse bending stresses for plates of various aspect ratios and axial load ratios are shown in the figures. Note that for large $\frac{a}{b}$ the normal displacement and stresses on lines $y = constant$ are nearly uniform at a distance from the ends greater than the width of the plate. Note that as the axial load ratio $\frac{F}{F_{cr}}$ is varied from positive to negative, the waviness of the plate near the ends increases.

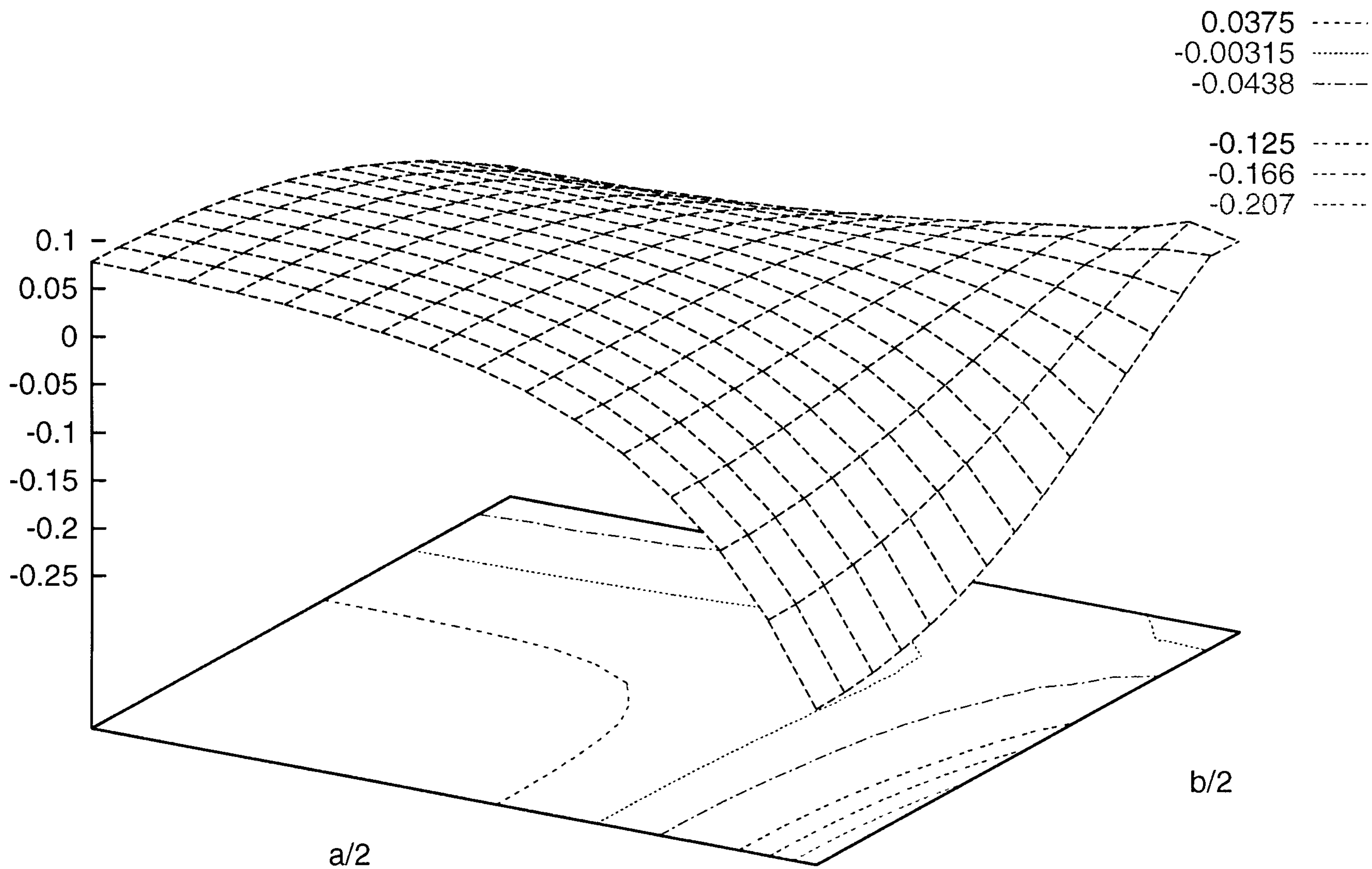
Maximum normal displacements, and maximum and minimum values of the longitudinal and transverse bending stresses are shown in separate graphs. The location of the maximum longitudinal bending stresses is always on the centerline $y = 0$, while the minimum longitudinal bending stresses are always at the endpoints $x = \pm\frac{a}{2}, y = 0$. The maximum transverse bending stresses are always on the centerline $y = 0$, while the minimum transverse bending stresses are always on the edgelines $y = \pm\frac{b}{2}$. Note that as the axial load ratio $\frac{F}{F_{cr}}$ is varied from positive to negative, the maximum stresses increase and the minimum stresses usually decrease. Also, note that for most cases the minimum transverse bending stress is the greatest stress in absolute value.

All values of $\frac{F}{F_{cr}}$ were greater than the buckling loads, given in the following table:

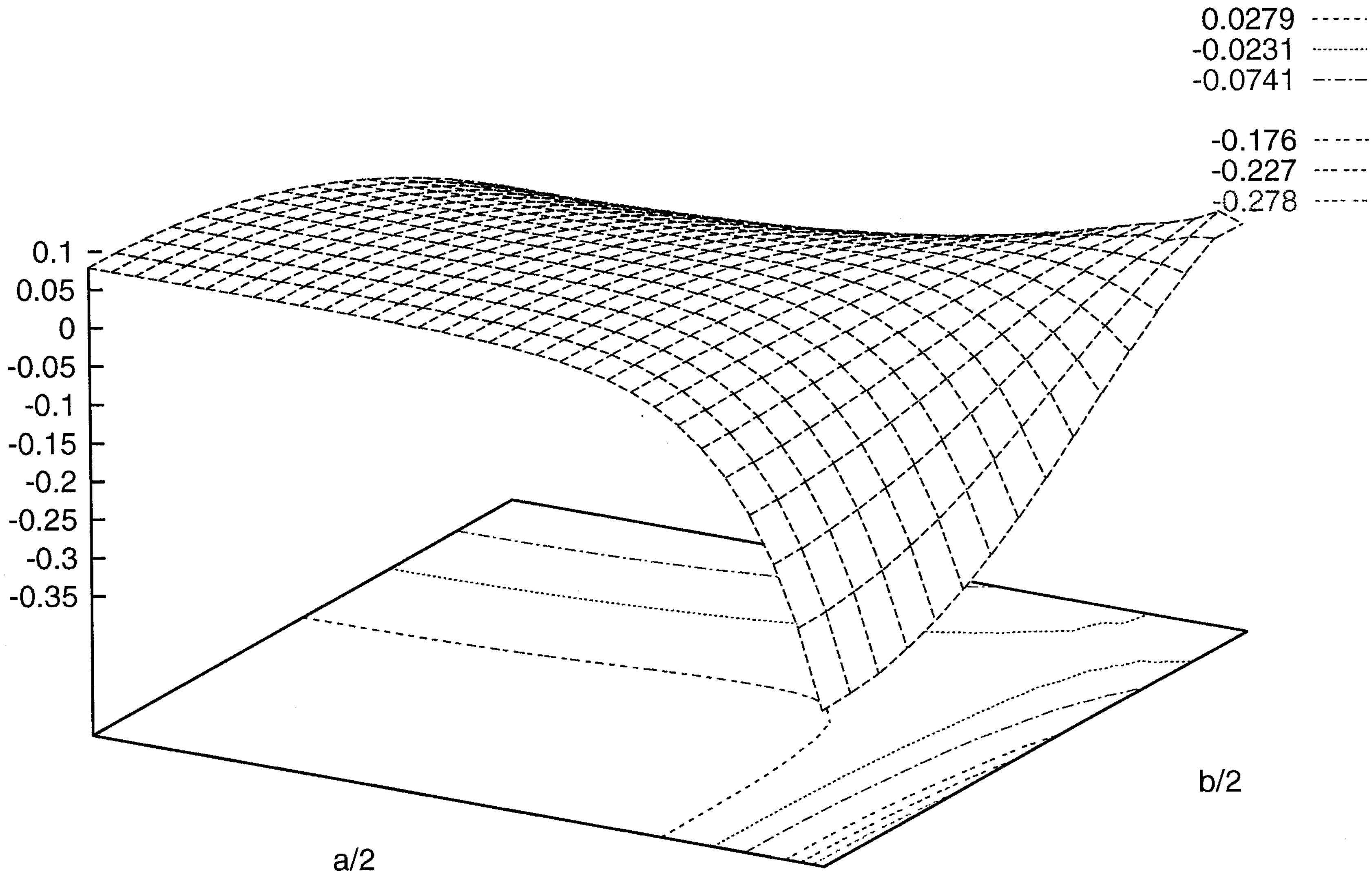
a/b	1	1.2	1.4	1.6	2	4	∞
Buckling Load F/F_{cr}	-2.52	-2.45	-2.41	-2.22	-1.99	-1.81	-1.75

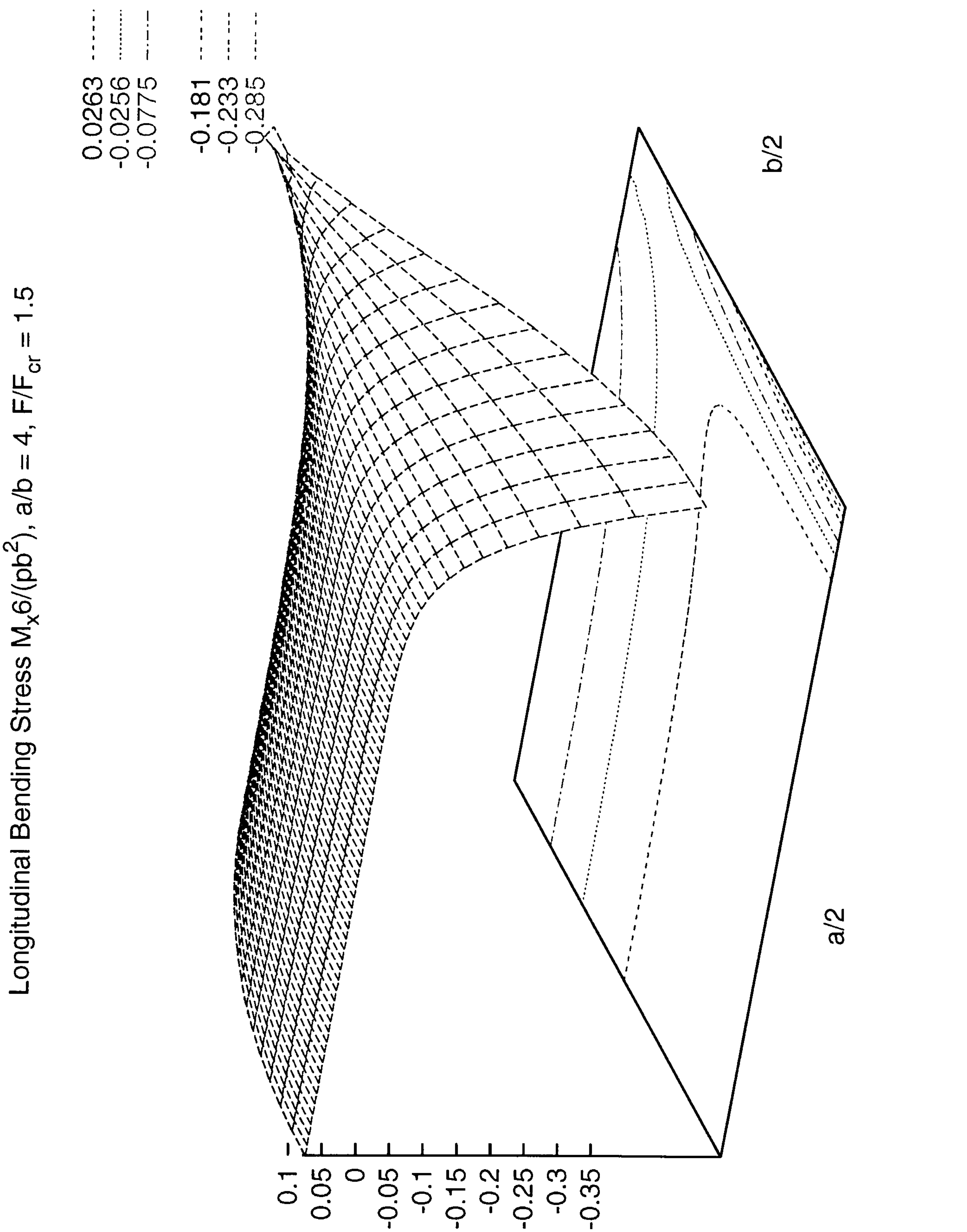
As a check on these results, the maximum normal displacements for the case $\frac{F}{F_{cr}} = 0$ (located at the origin $x = y = 0$) were checked against Roark and Young (1975), the transverse bending stress distribution on the line $x = 0$ was verified to be nearly parabolic for large $\frac{a}{b}$, and the buckling loads were checked against Timoshenko and Gere (1961).

Longitudinal Bending Stress $M_x 6/(pb^2)$, $a/b = 1$, $F/F_{cr} = 1.5$



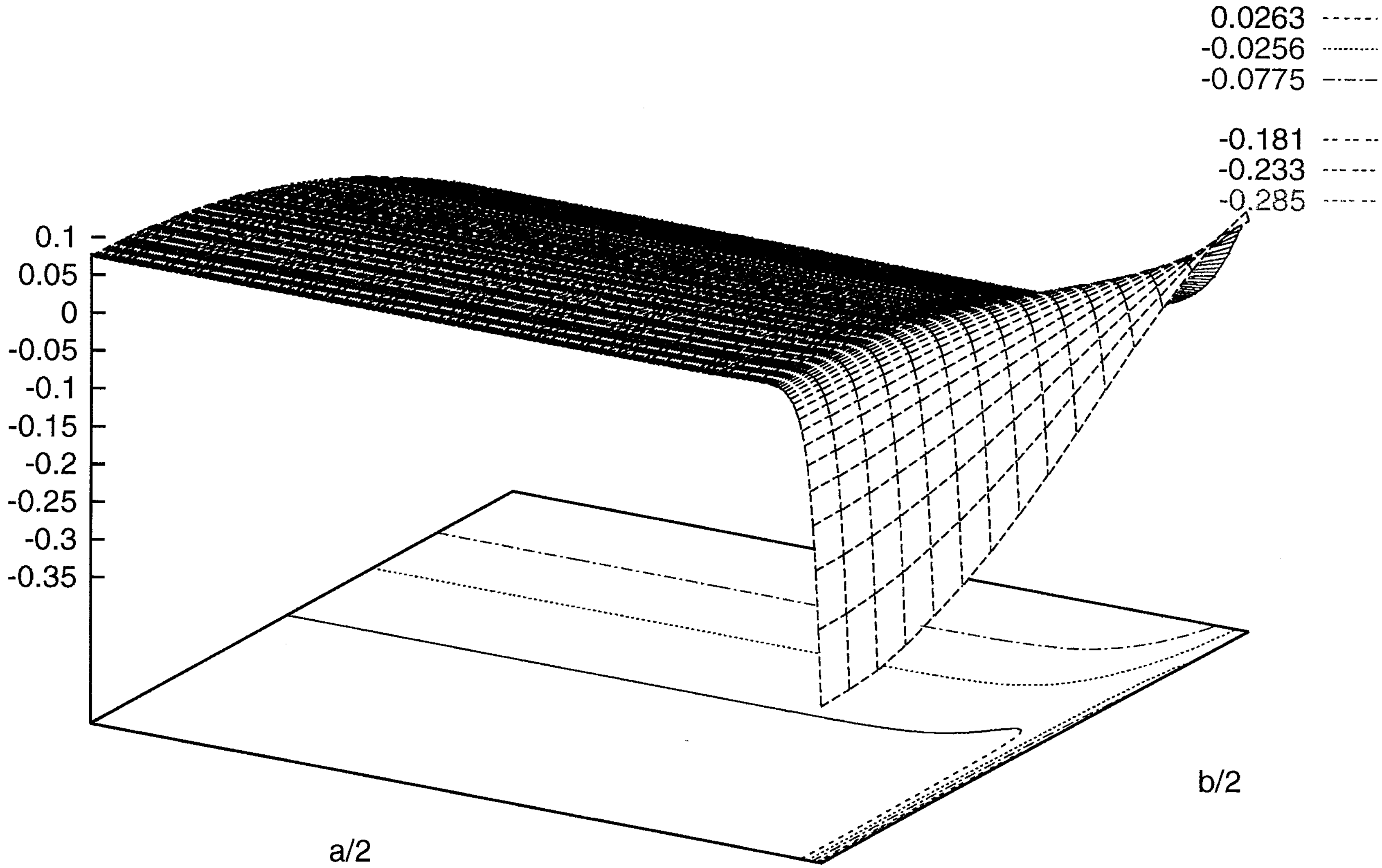
Longitudinal Bending Stress $M_x 6/(pb^2)$, a/b = 2, F/F_{cr} = 1.5

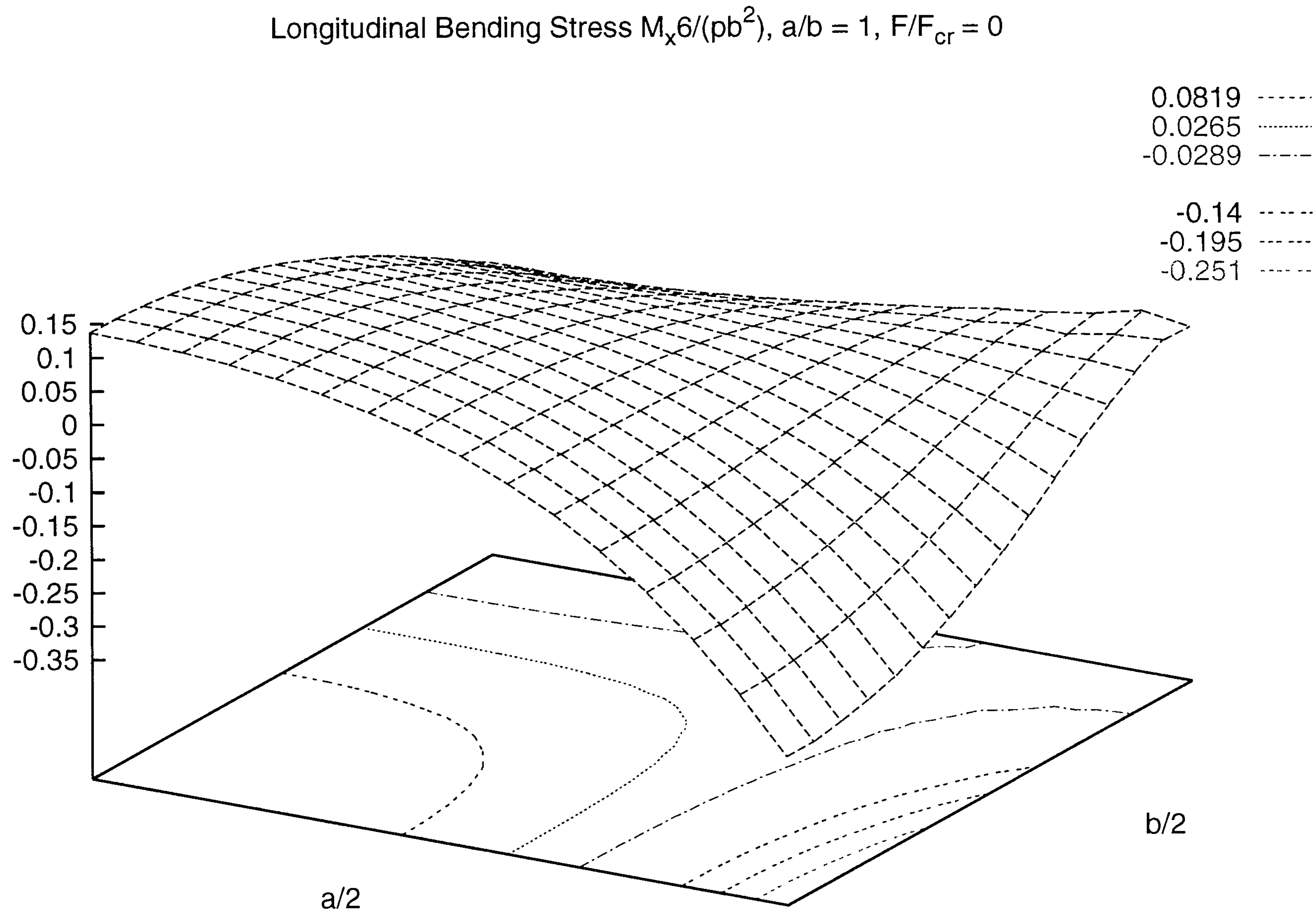


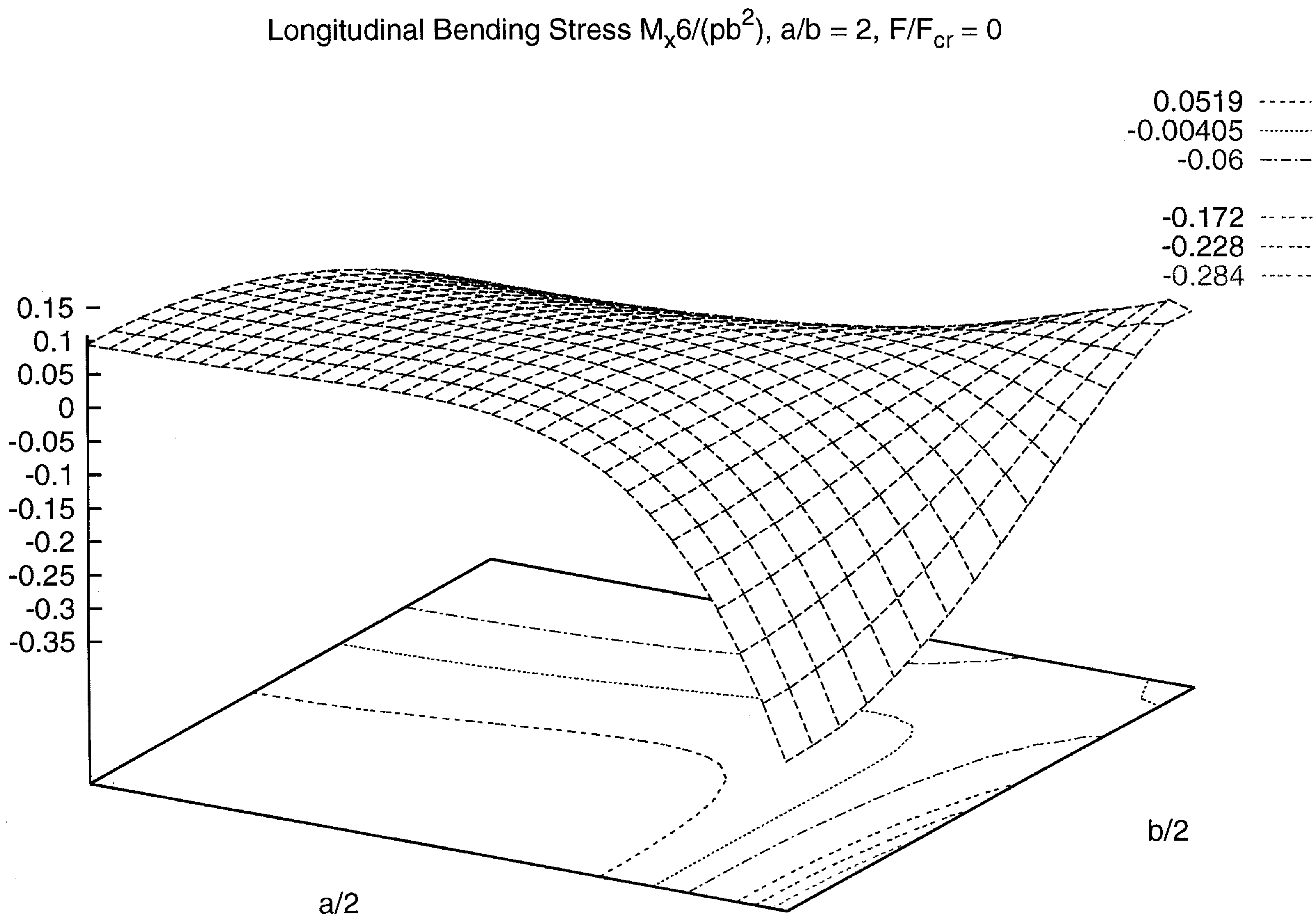


Longitudinal Bending Stress $M_x 6/(pb^2)$, a/b = 4, F/F_{cr} = 1.5
0.0263
-0.0256
-0.0775
-0.181
-0.233
-0.285
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.35
a/2
b/2

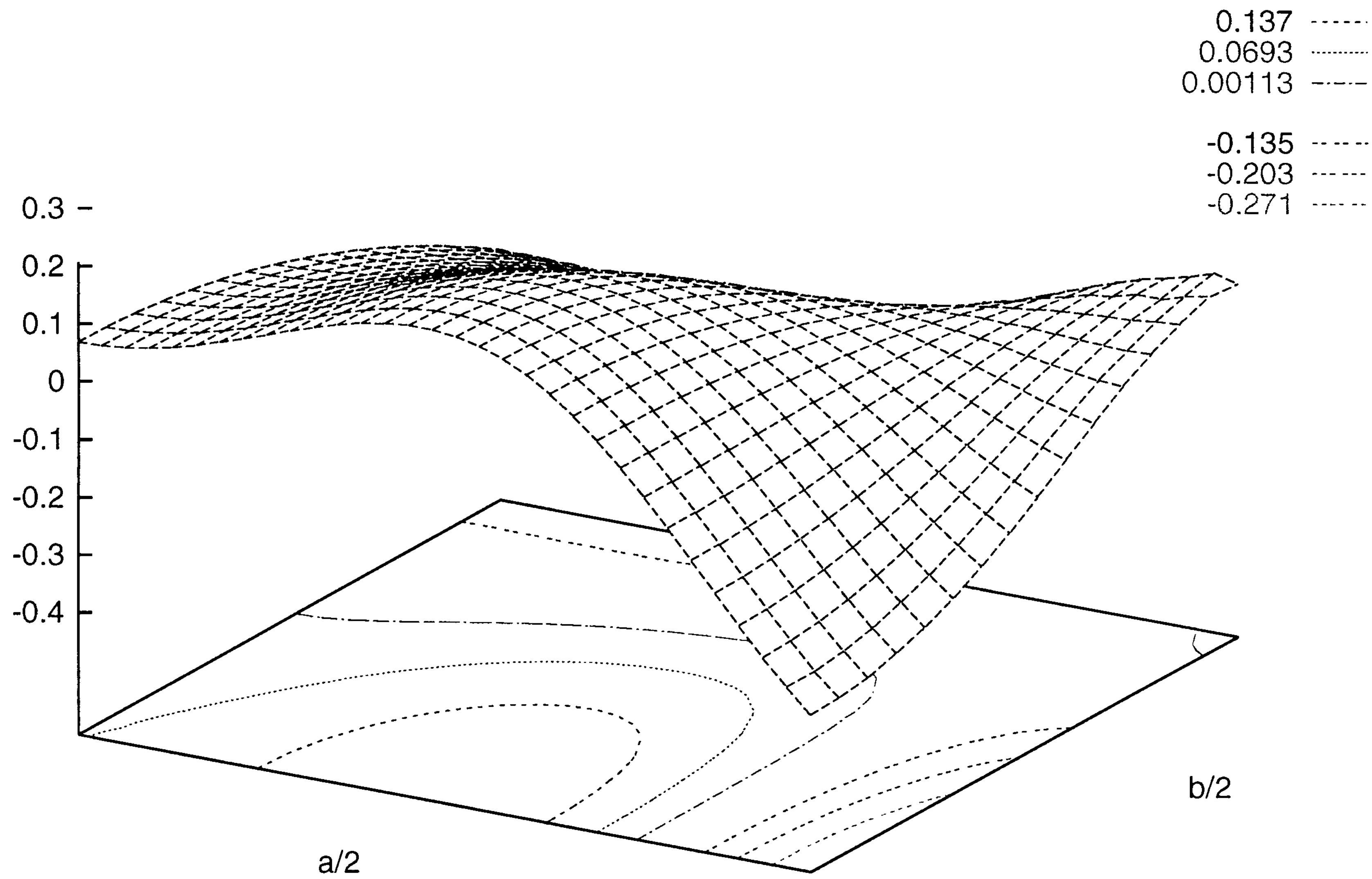
Longitudinal Bending Stress $M_x 6/(pb^2)$, a/b = 16, F/F_{cr} = 1.5



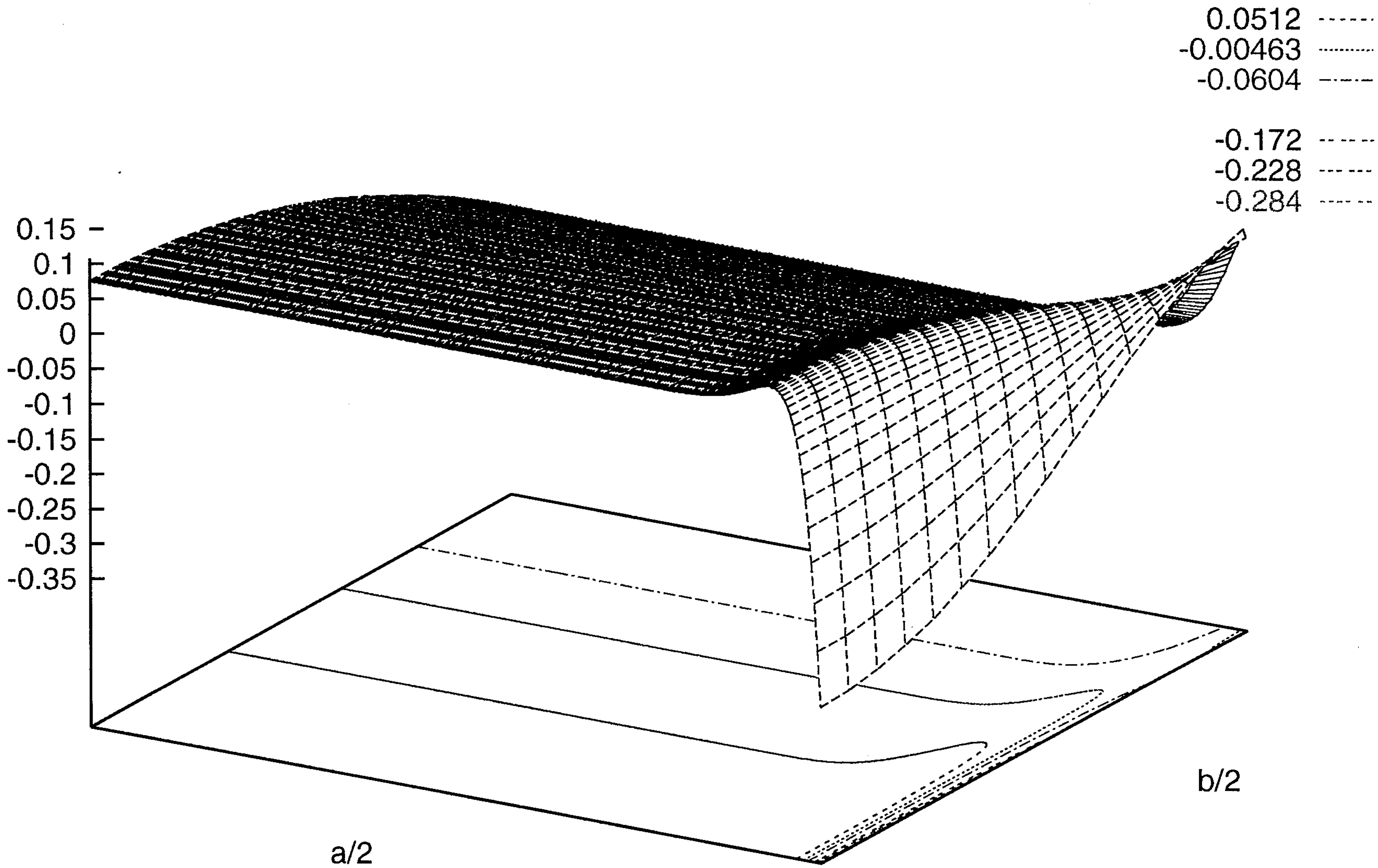

Longitudinal Bending Stress $M_x 6/(pb^2)$, a/b = 1, F/F_{cr} = 0
0.0819
0.0265
-0.0289
-0.14
-0.195
-0.251
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.35
a/2
b/2


Longitudinal Bending Stress $M_x 6/(pb^2)$, a/b = 2, F/F_{cr} = 0
0.0519
-0.00405
-0.06
-0.172
-0.228
-0.284
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.35
a/2
b/2

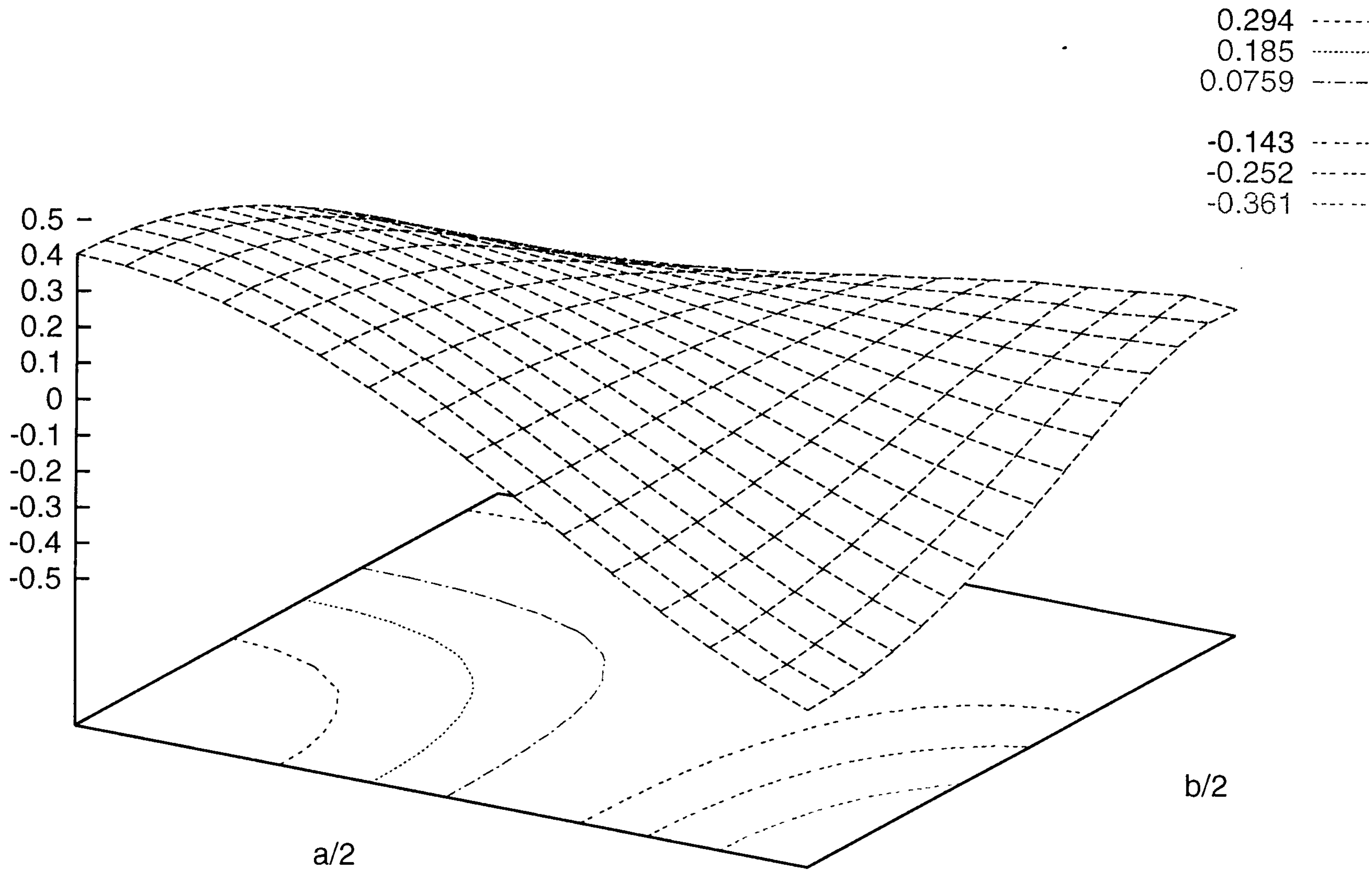
Longitudinal Bending Stress $M_x 6/(pb^2)$, $a/b = 4$, $F/F_{cr} = 0$



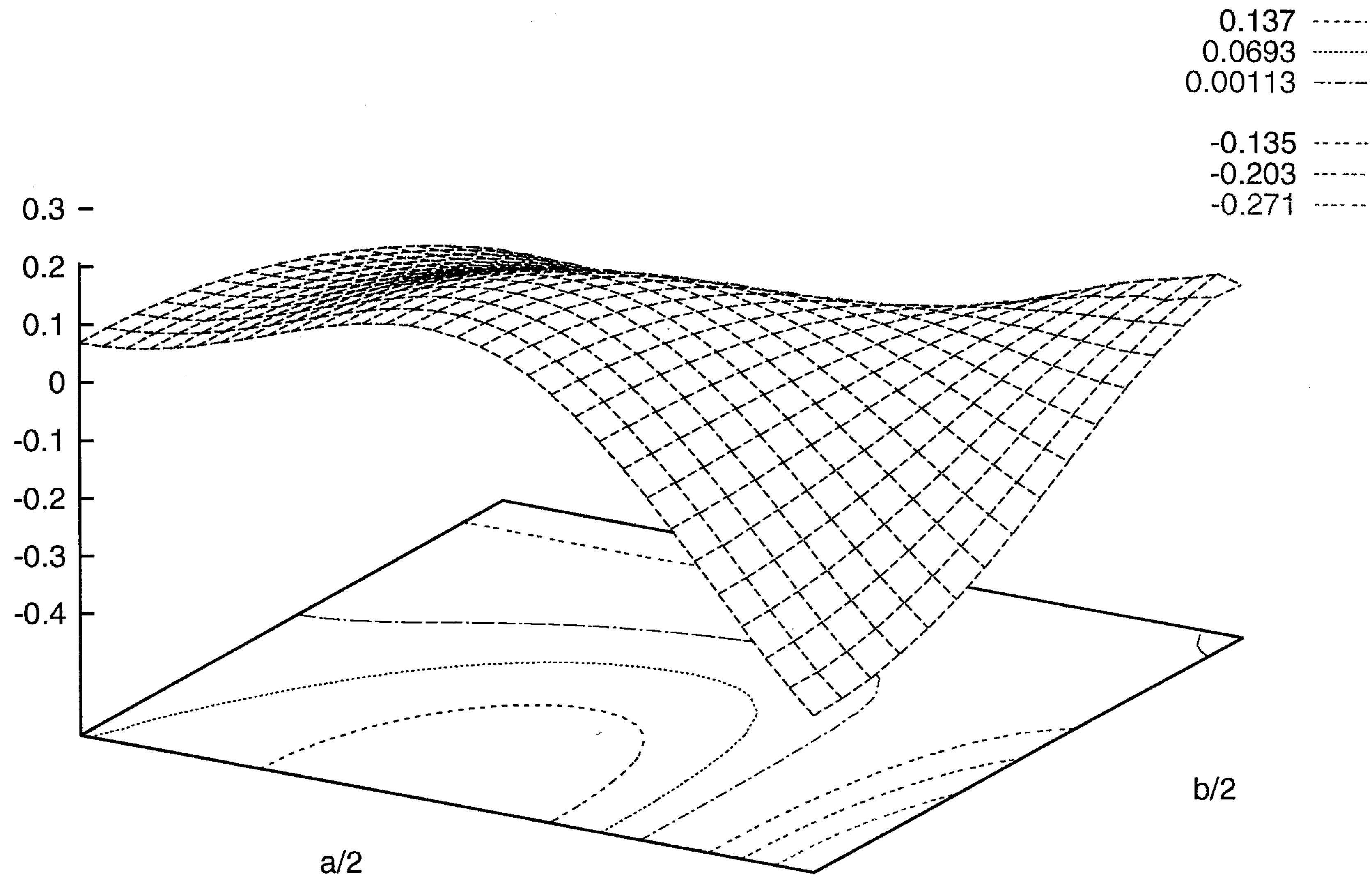
Longitudinal Bending Stress $M_x 6/(pb^2)$, $a/b = 16$, $F/F_{cr} = 0$



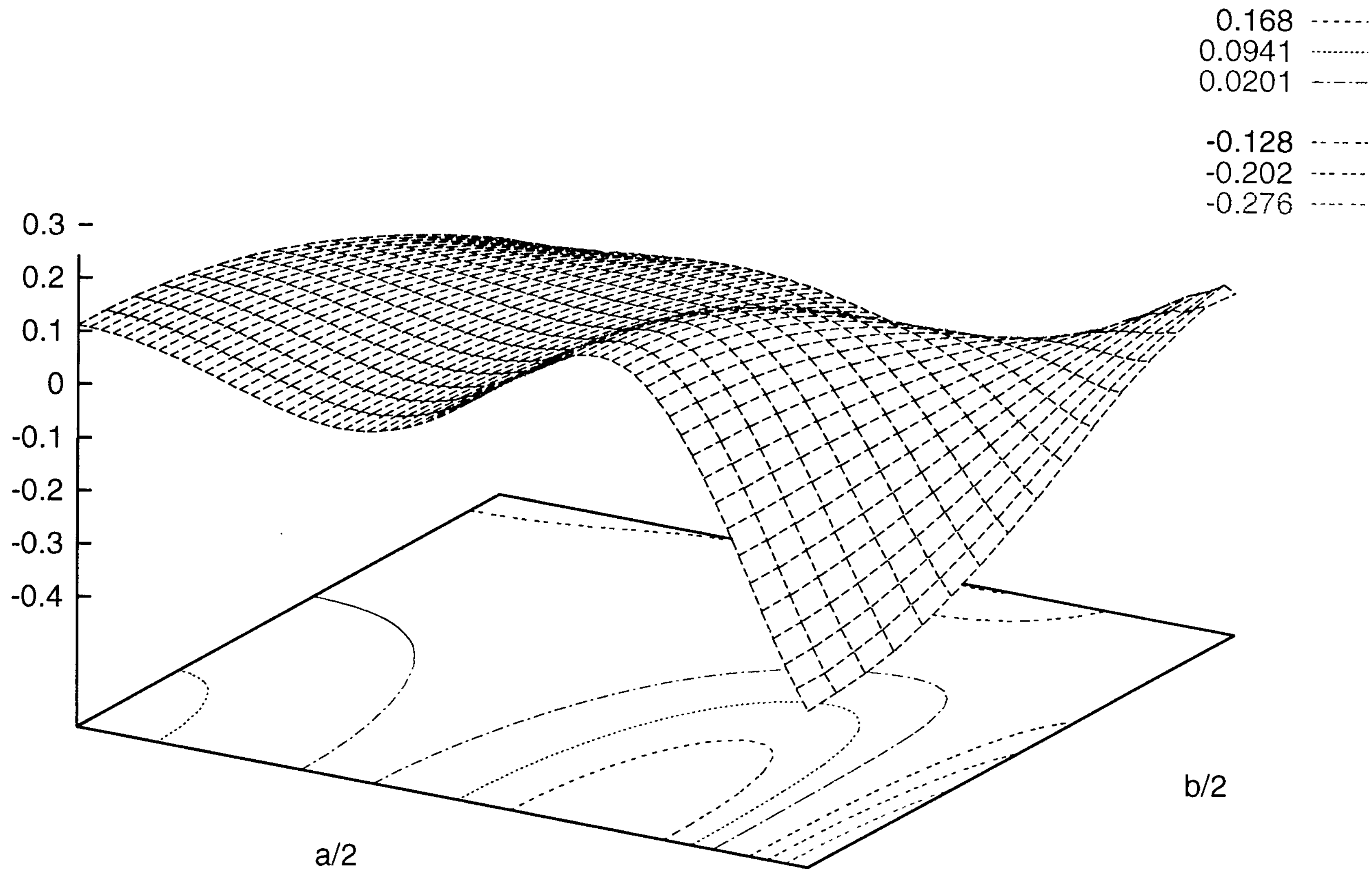
Longitudinal Bending Stress $M_x 6/(pb^2)$, $a/b = 1$, $F/F_{cr} = -1.5$



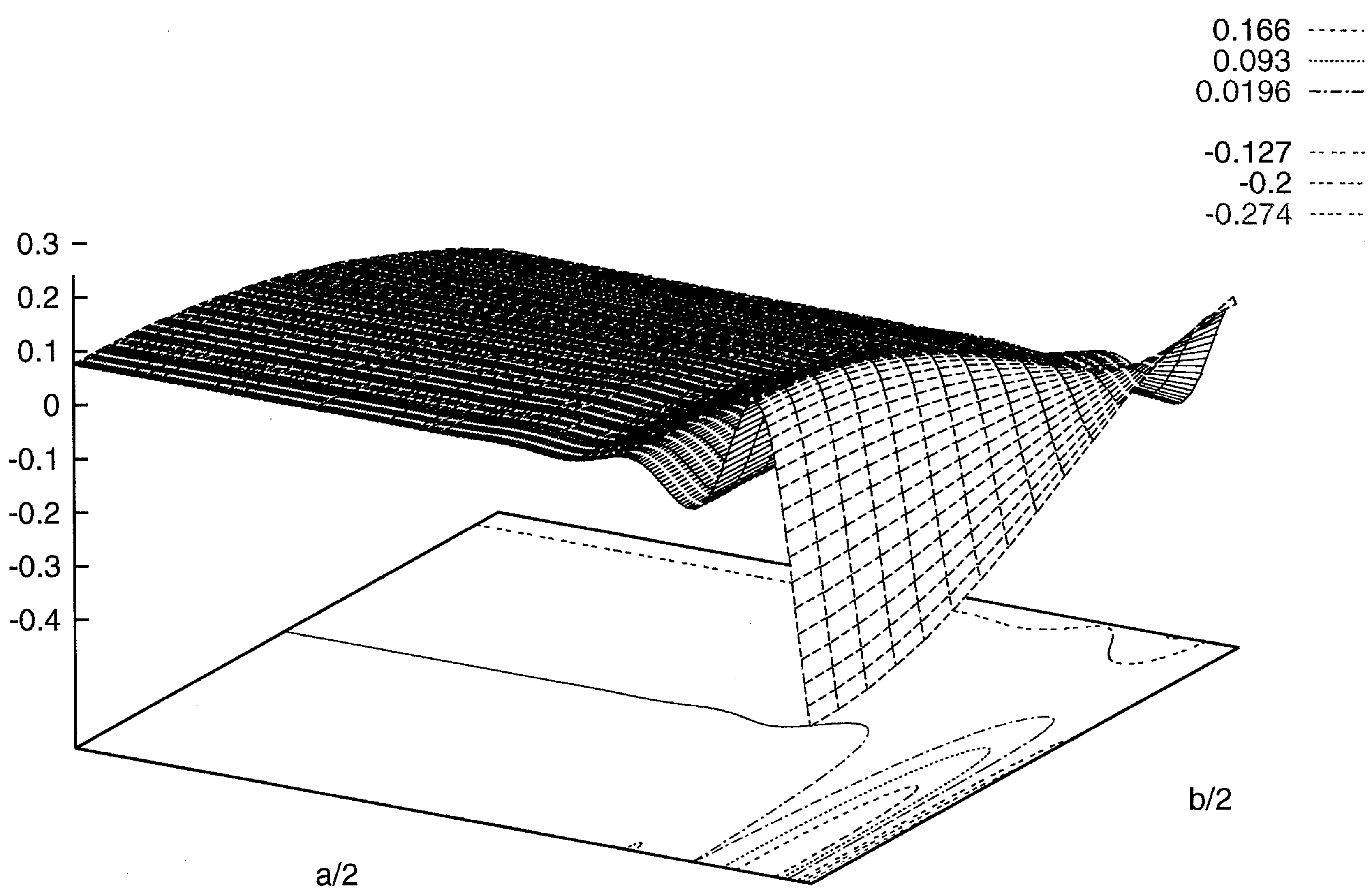
Longitudinal Bending Stress $M_x6/(pb^2)$, $a/b = 2$, $F/F_{cr} = -1.5$



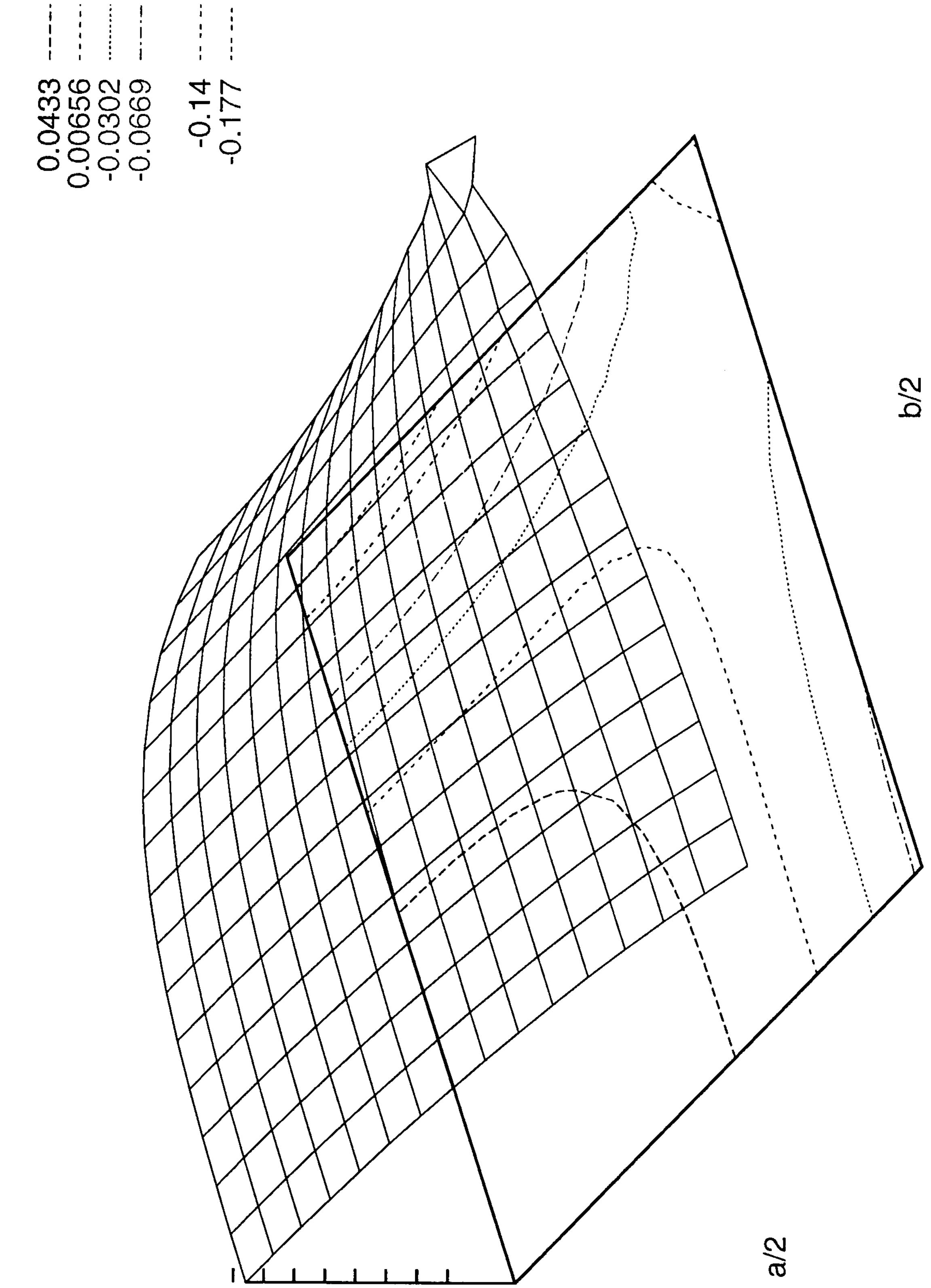
Longitudinal Bending Stress $M_x 6/(pb^2)$, a/b = 4, F/F_{cr} = -1.5



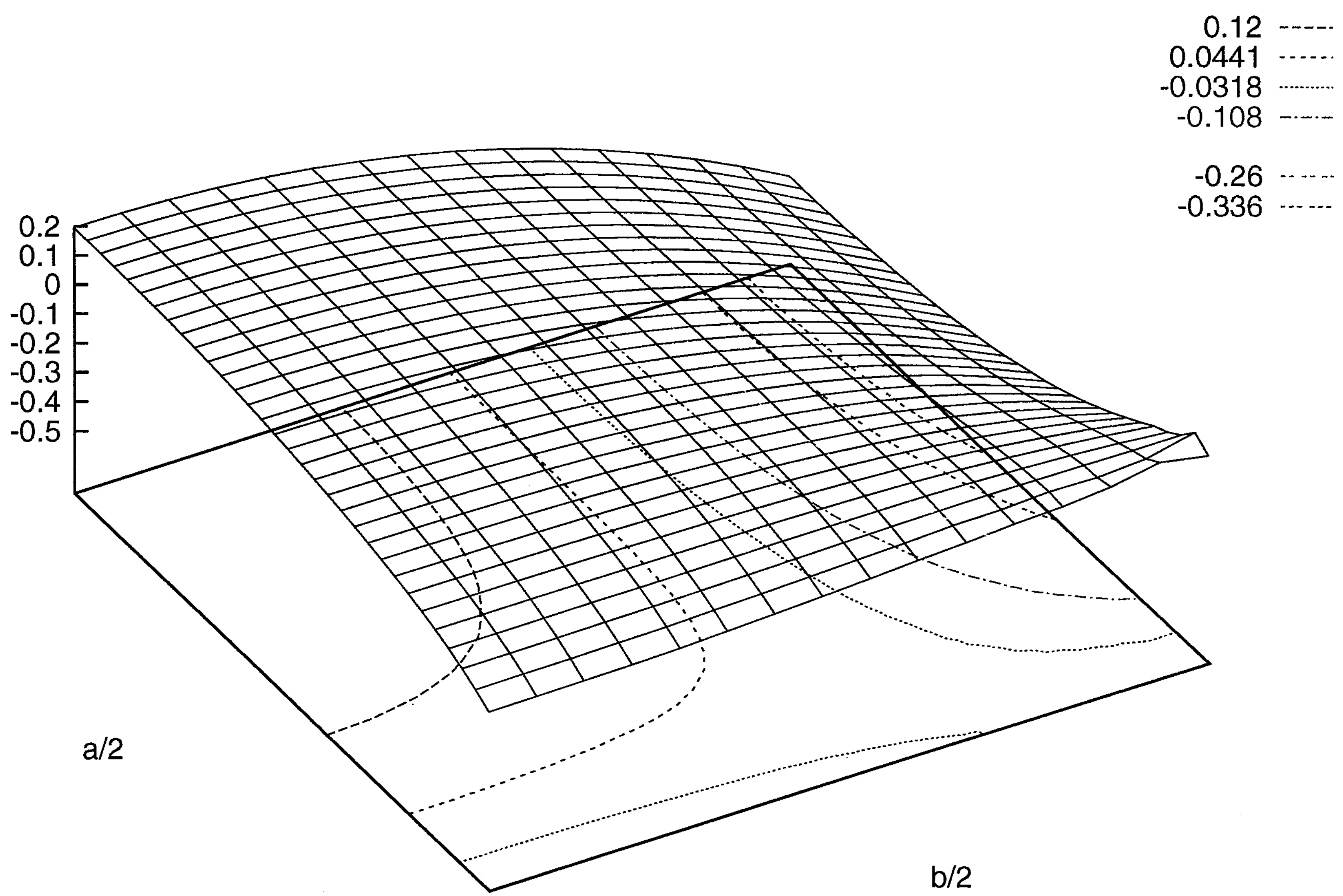
Longitudinal Bending Stress $M_x 6/(pb^2)$, a/b = 16, F/F_{cr} = -1.5



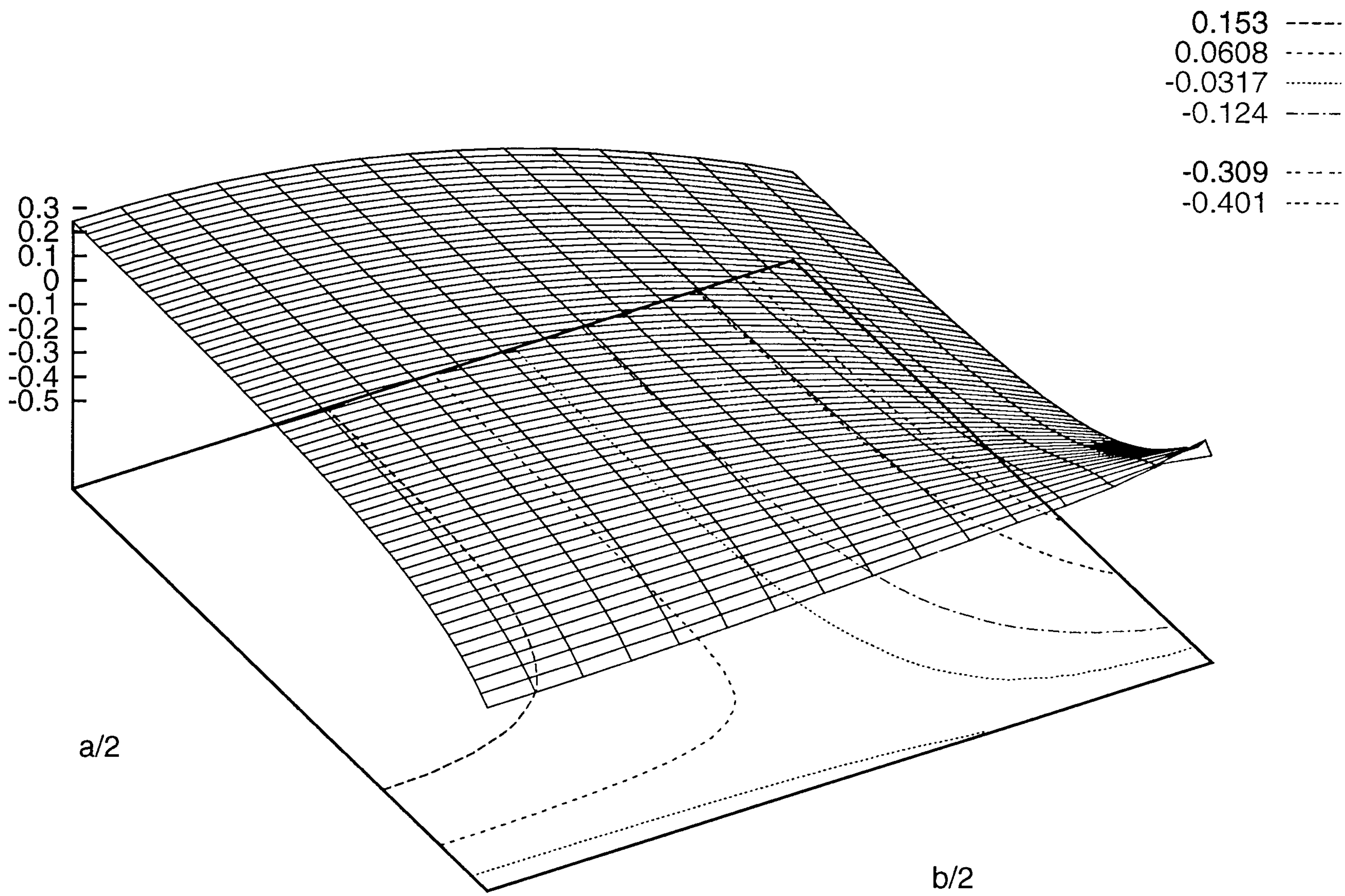
Transverse Bending Stress $M_y 6/(pb^2)$, $a/b = 1$, $F/F_{cr} = 1.5$



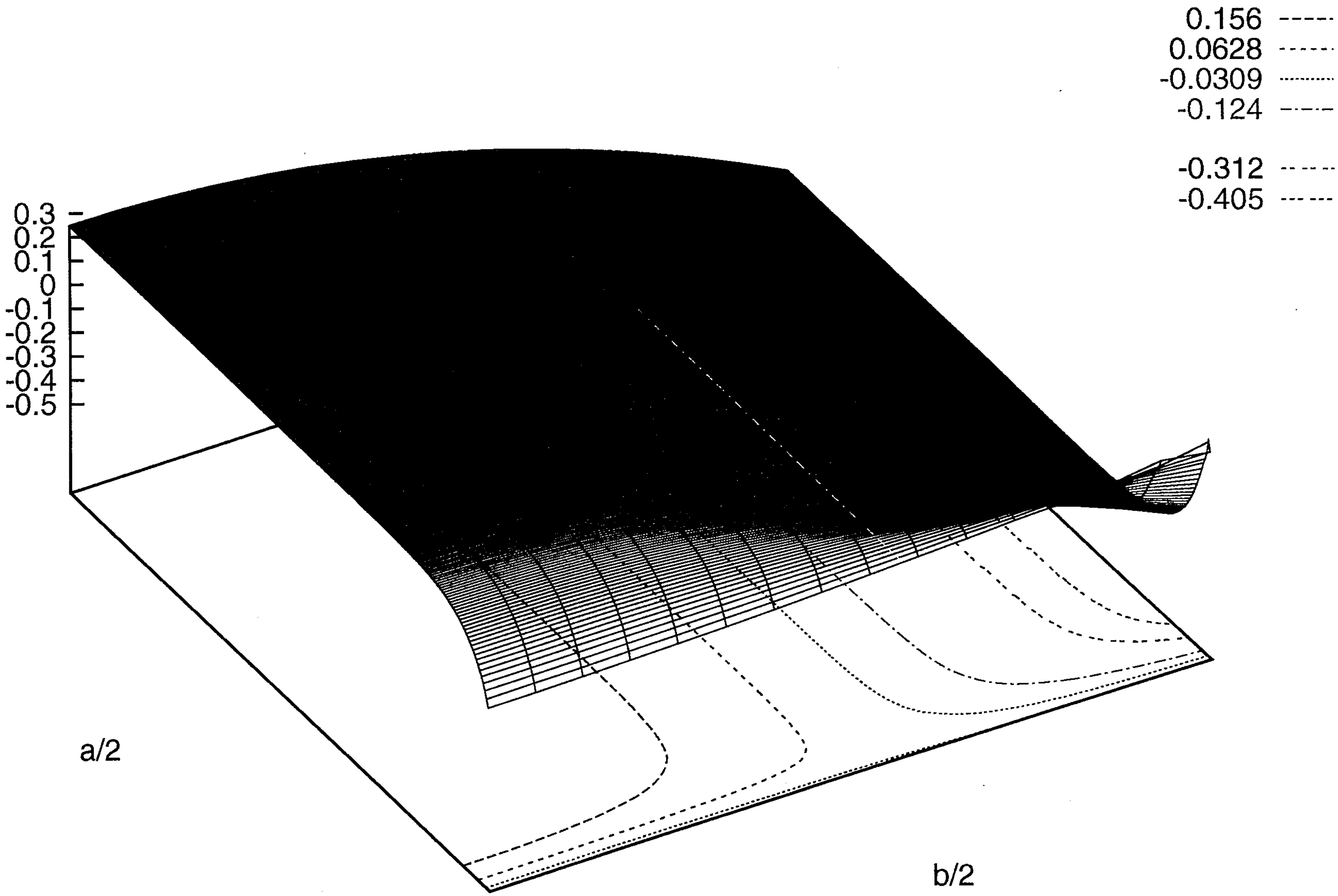
Transverse Bending Stress $M_y 6/(pb^2)$, a/b = 2, F/F_{cr} = 1.5



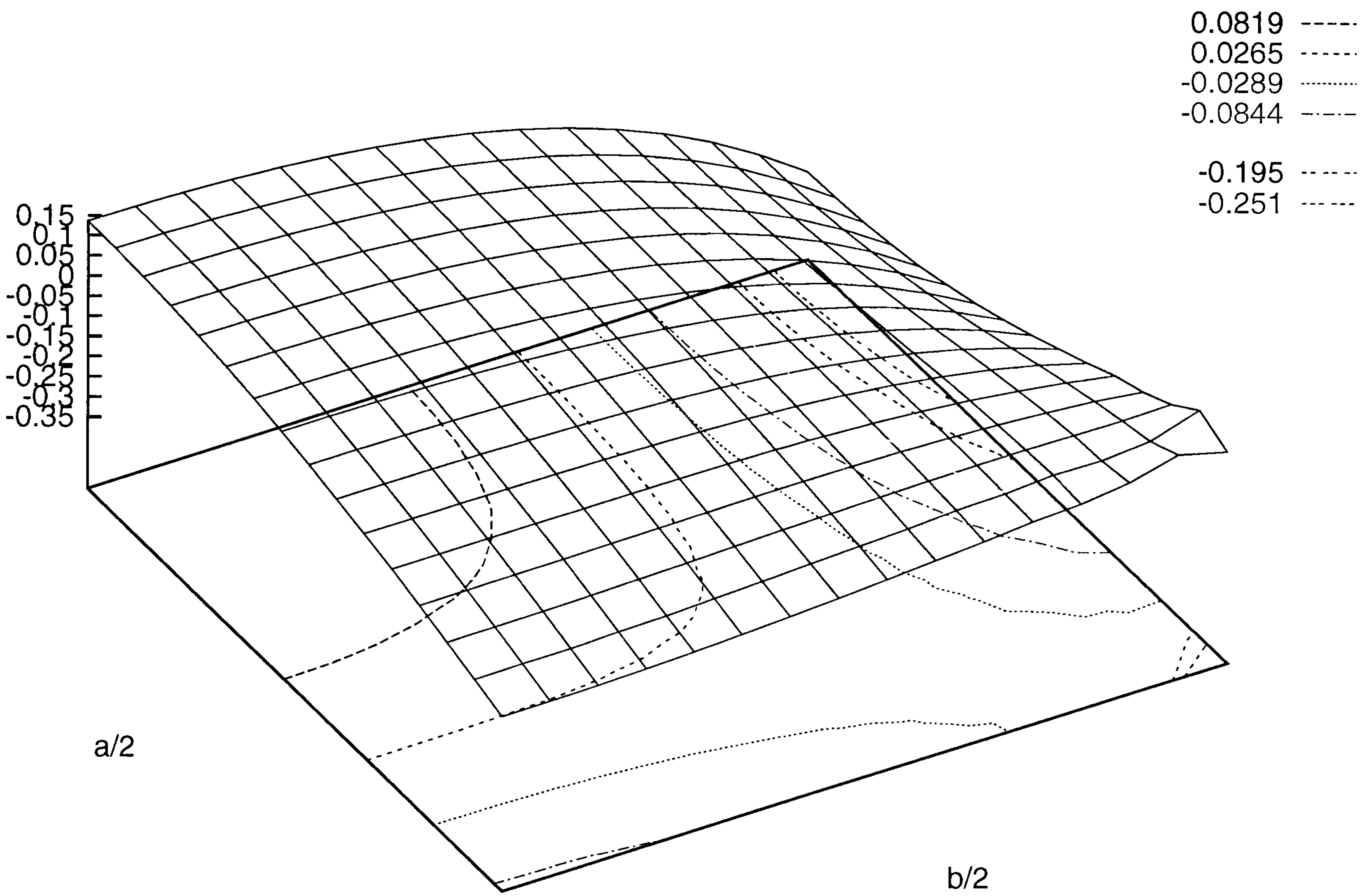
Transverse Bending Stress $M_y 6/(pb^2)$, $a/b = 4$, $F/F_{cr} = 1.5$



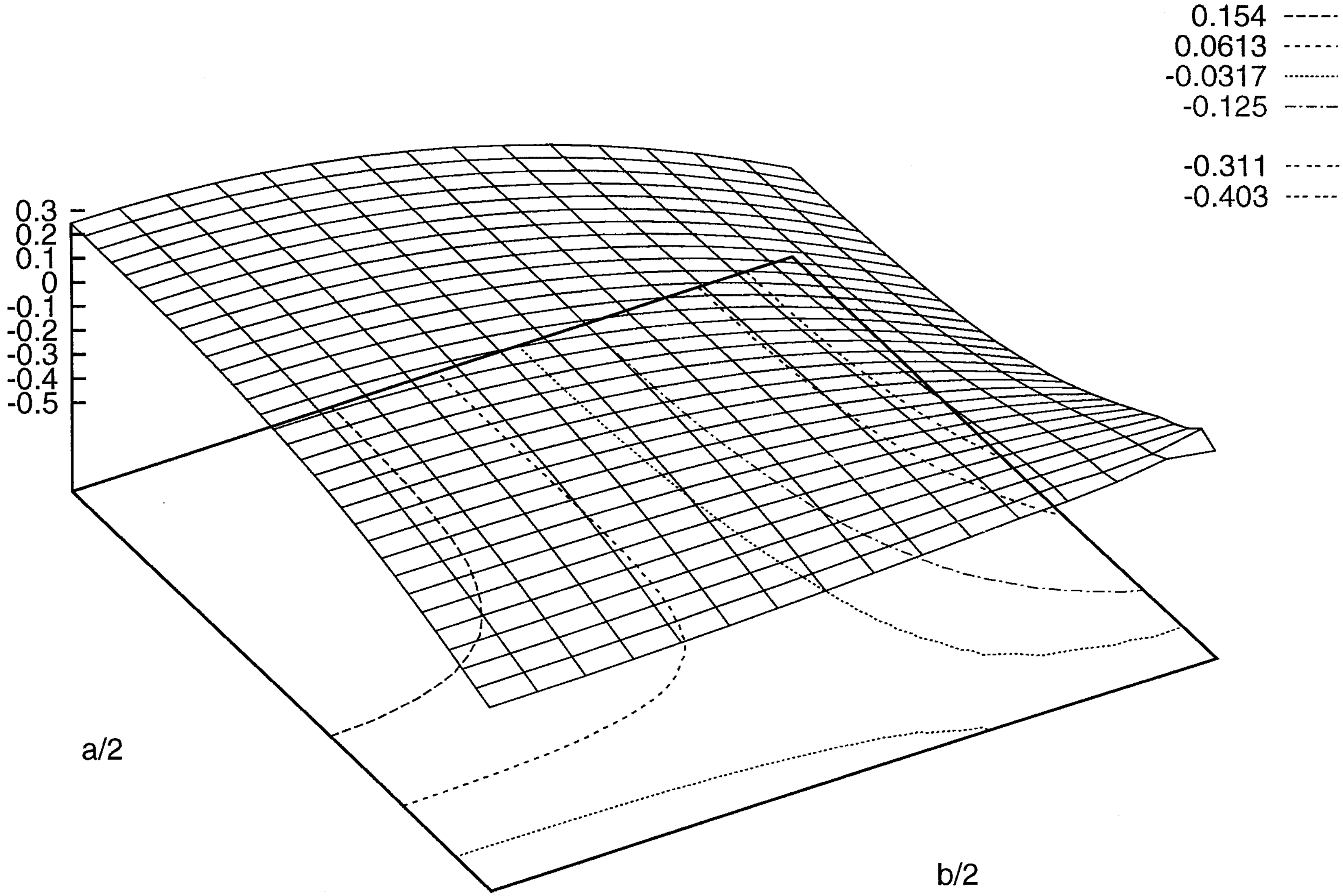
Transverse Bending Stress $M_y 6/(pb^2)$, $a/b = 16$, $F/F_{cr} = 1.5$



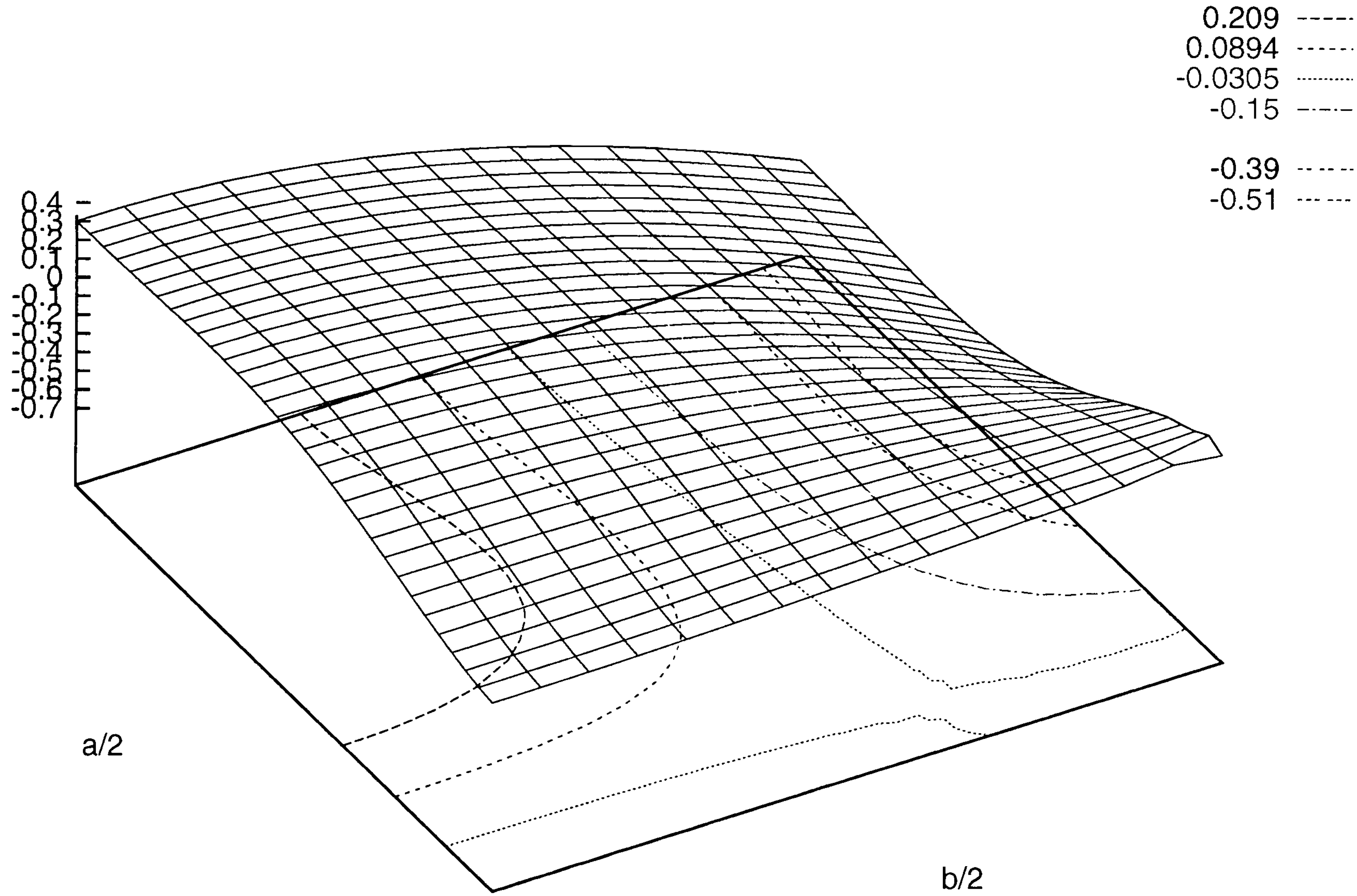
Transverse Bending Stress $M_y 6/(pb^2)$, $a/b = 1$, $F/F_{cr} = 0$



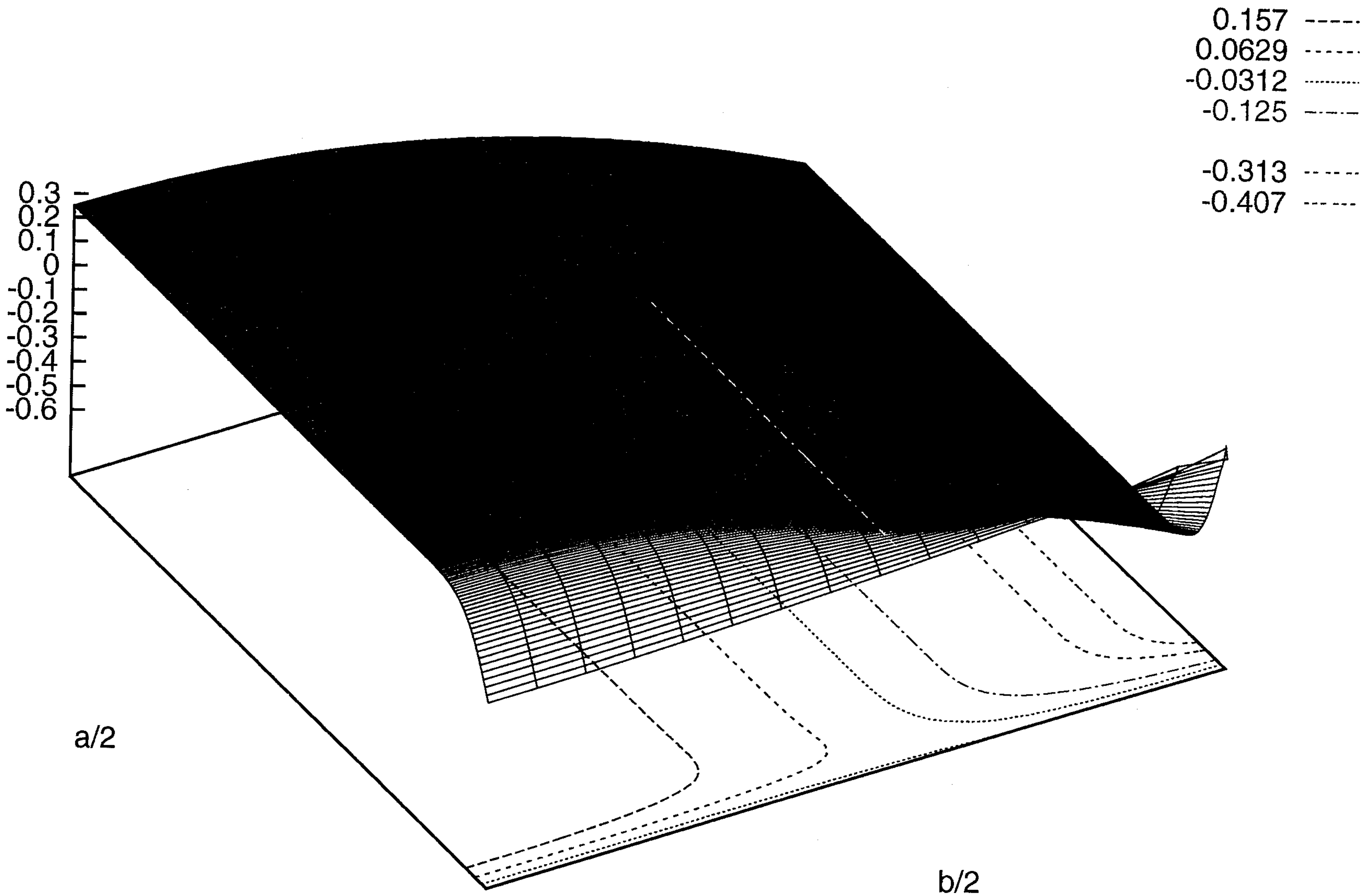
Transverse Bending Stress $M_y 6/(pb^2)$, a/b = 2, $F/F_{cr} = 0$

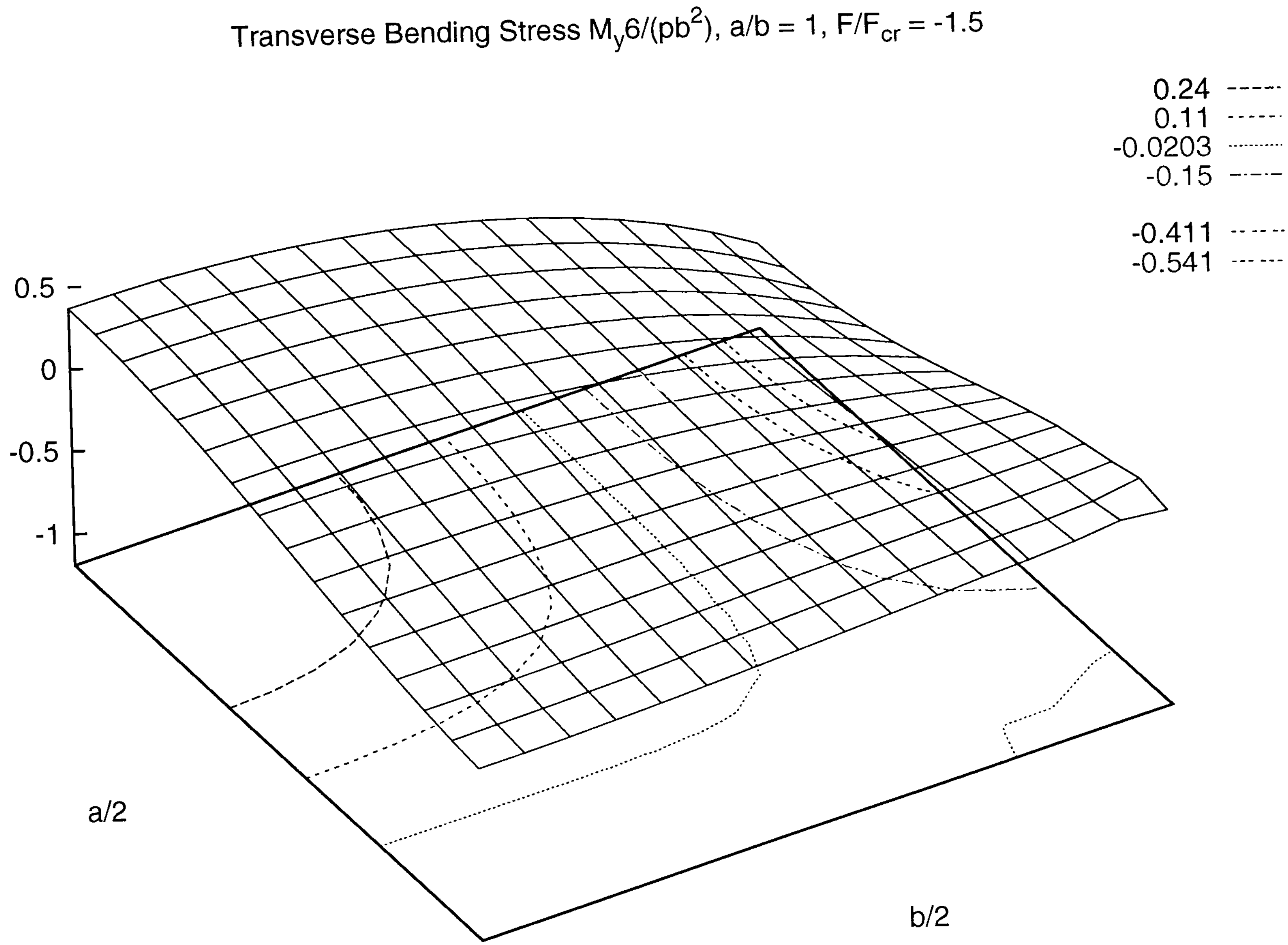


Transverse Bending Stress $M_y 6/(pb^2)$, $a/b = 4$, $F/F_{cr} = 0$

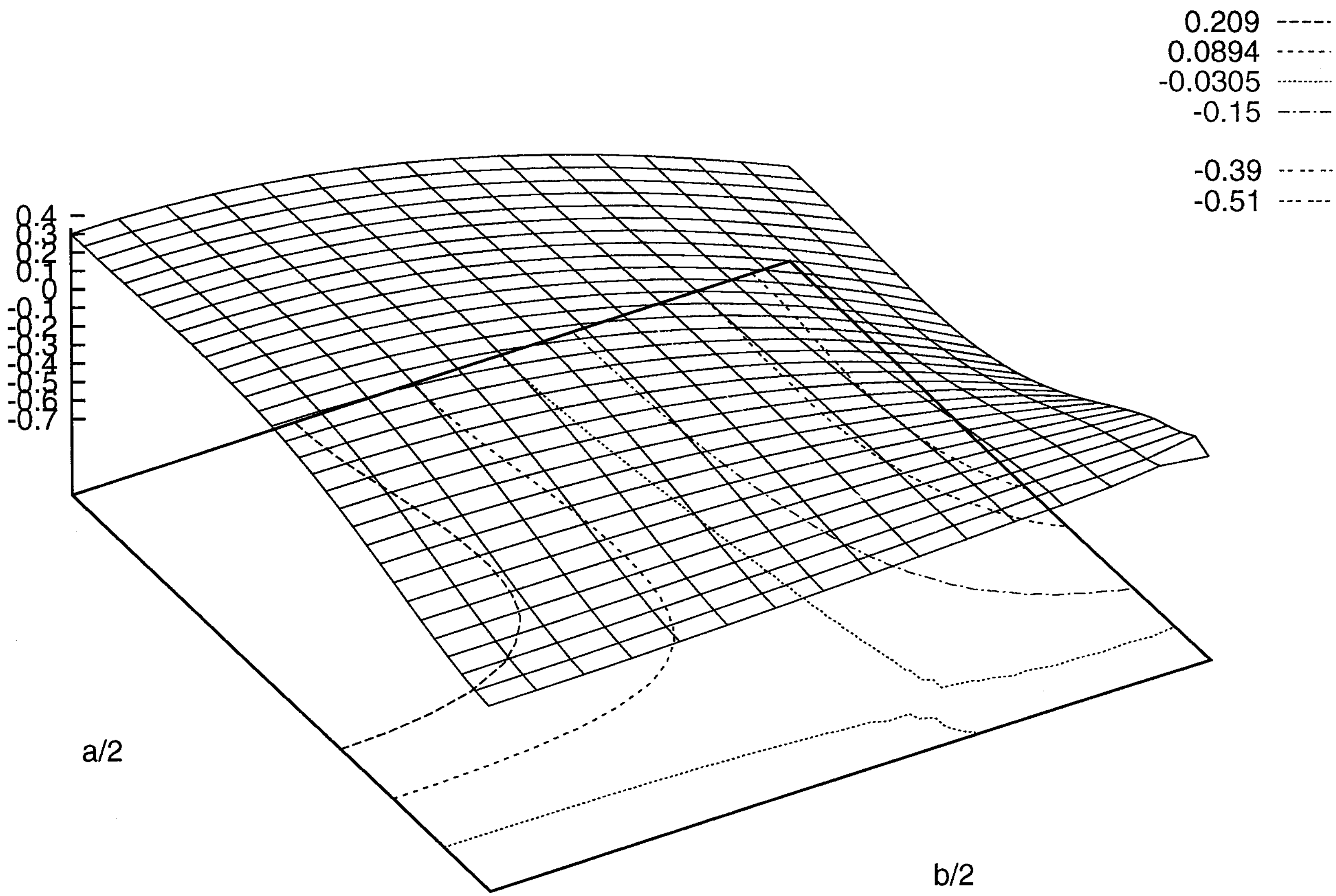


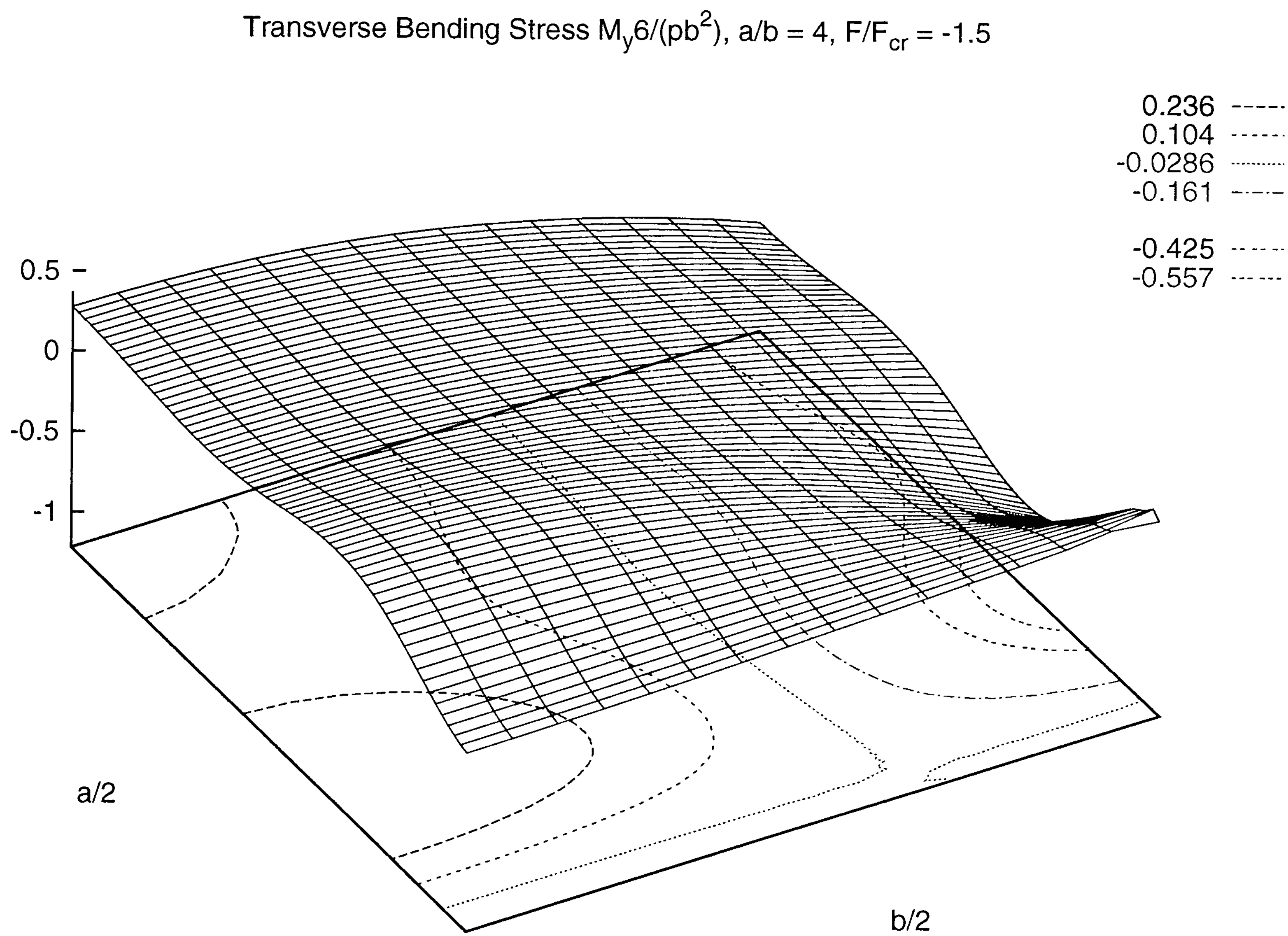
Transverse Bending Stress $M_y 6/(pb^2)$, $a/b = 16$, $F/F_{cr} = 0$



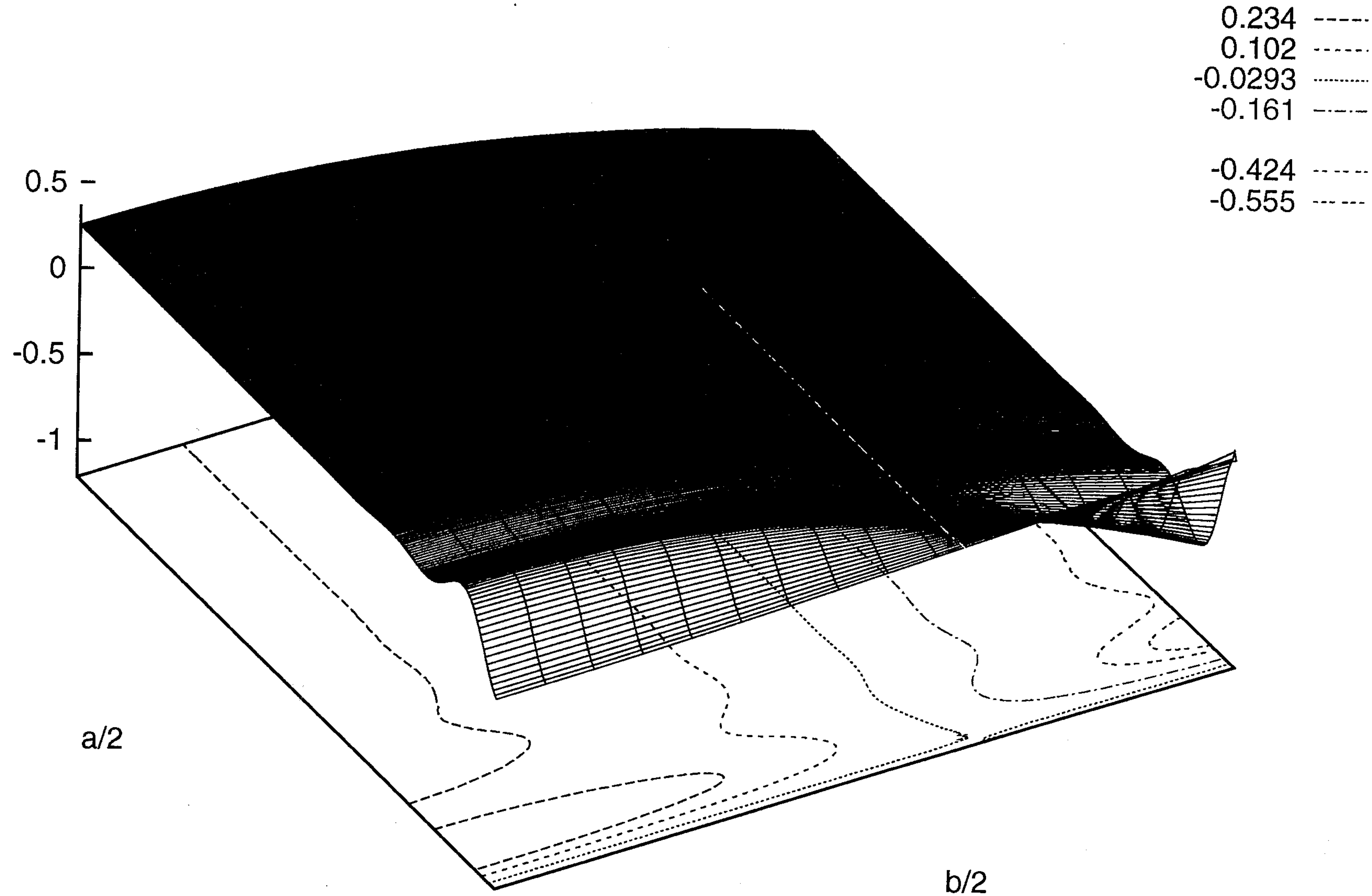

Transverse Bending Stress $M_y 6/(pb^2)$, a/b = 1, F/F_{cr} = -1.5
0.24
0.11
-0.0203
-0.15
-0.411
-0.541
0.5
0
-0.5
-1
a/2
b/2

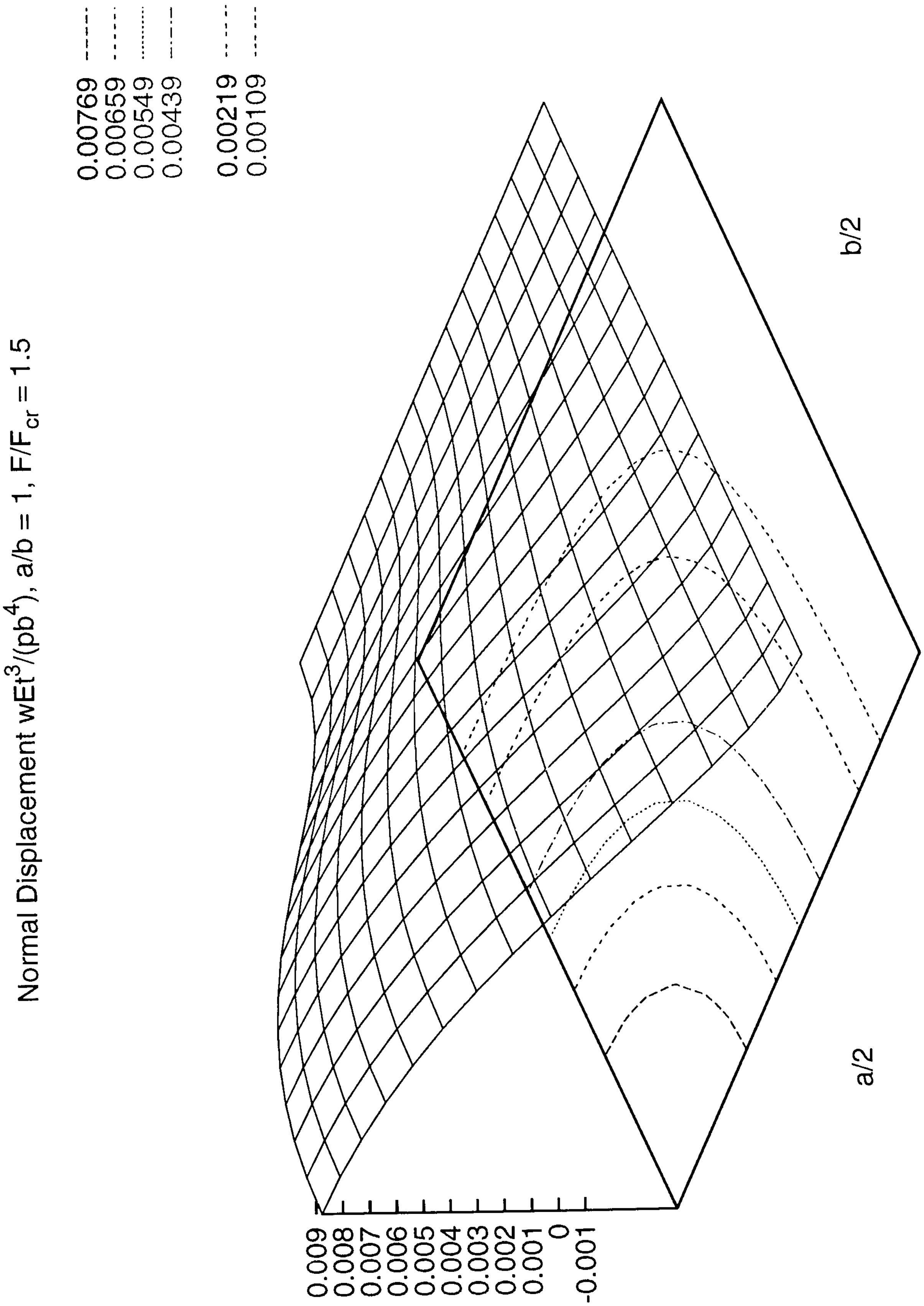
Transverse Bending Stress $M_y 6/(pb^2)$, $a/b = 2$, $F/F_{cr} = -1.5$



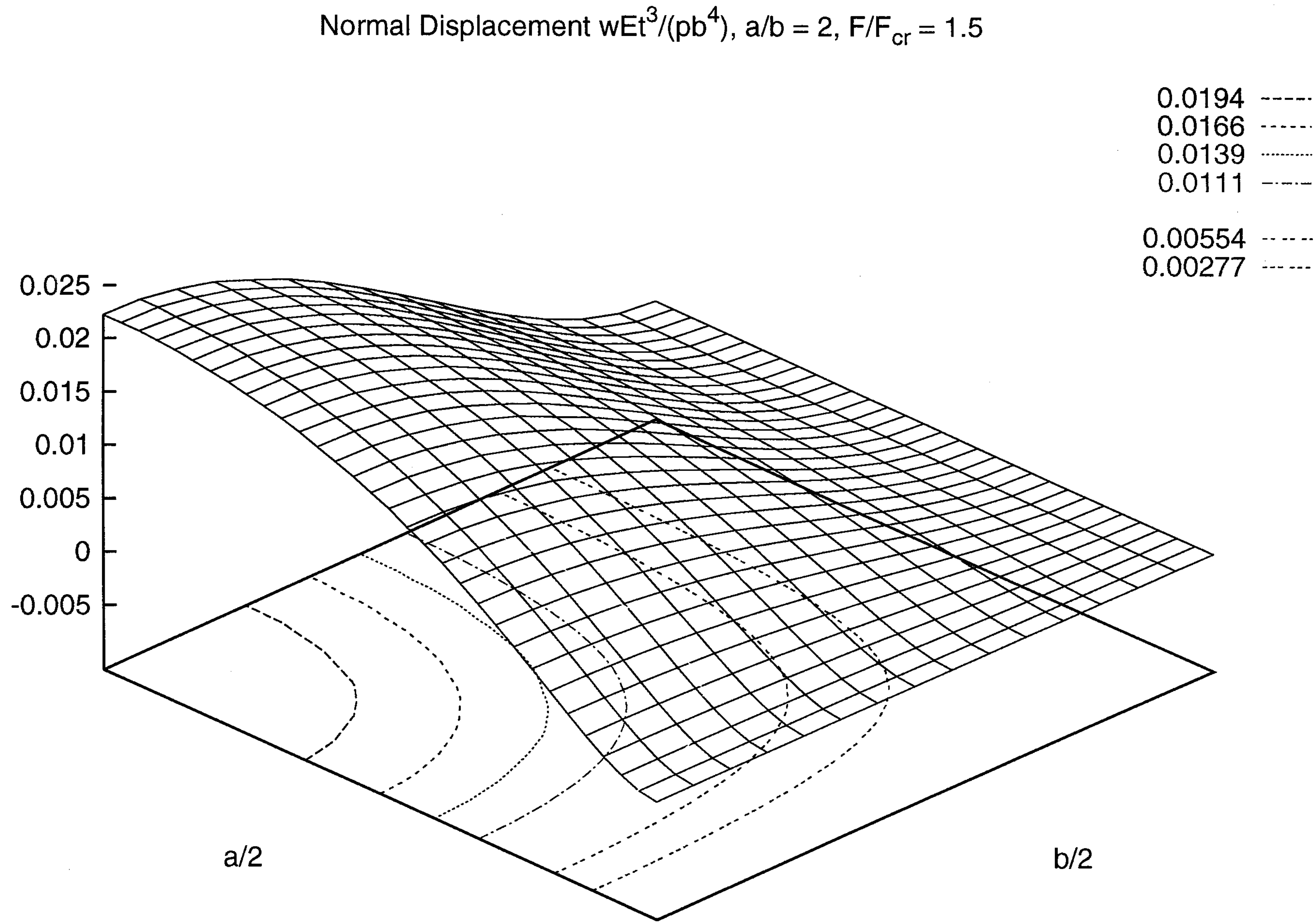

Transverse Bending Stress M_y6/(pb^2), a/b = 4, F/F_cr = -1.5
0.236
0.104
-0.0286
-0.161
-0.425
-0.557
0.5
0
-0.5
-1
a/2
b/2

Transverse Bending Stress $M_y 6/(pb^2)$, $a/b = 16$, $F/F_{cr} = -1.5$



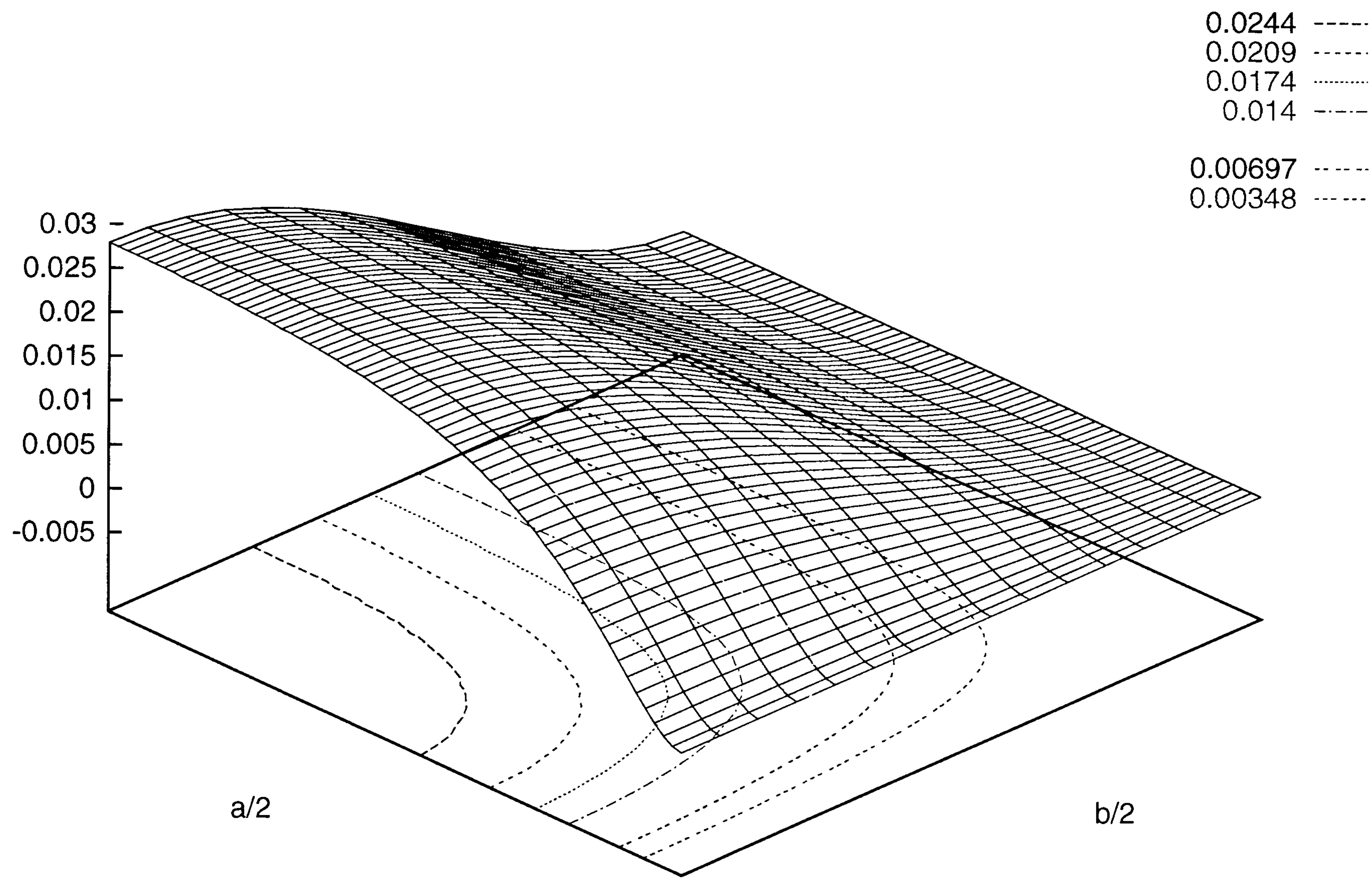


Normal Displacement $wEt^3/(pb^4)$, $a/b = 1$, $F/F_{cr} = 1.5$
0.00769
0.00659
0.00549
0.00439
0.00219
0.00109
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
-0.001
a/2
b/2

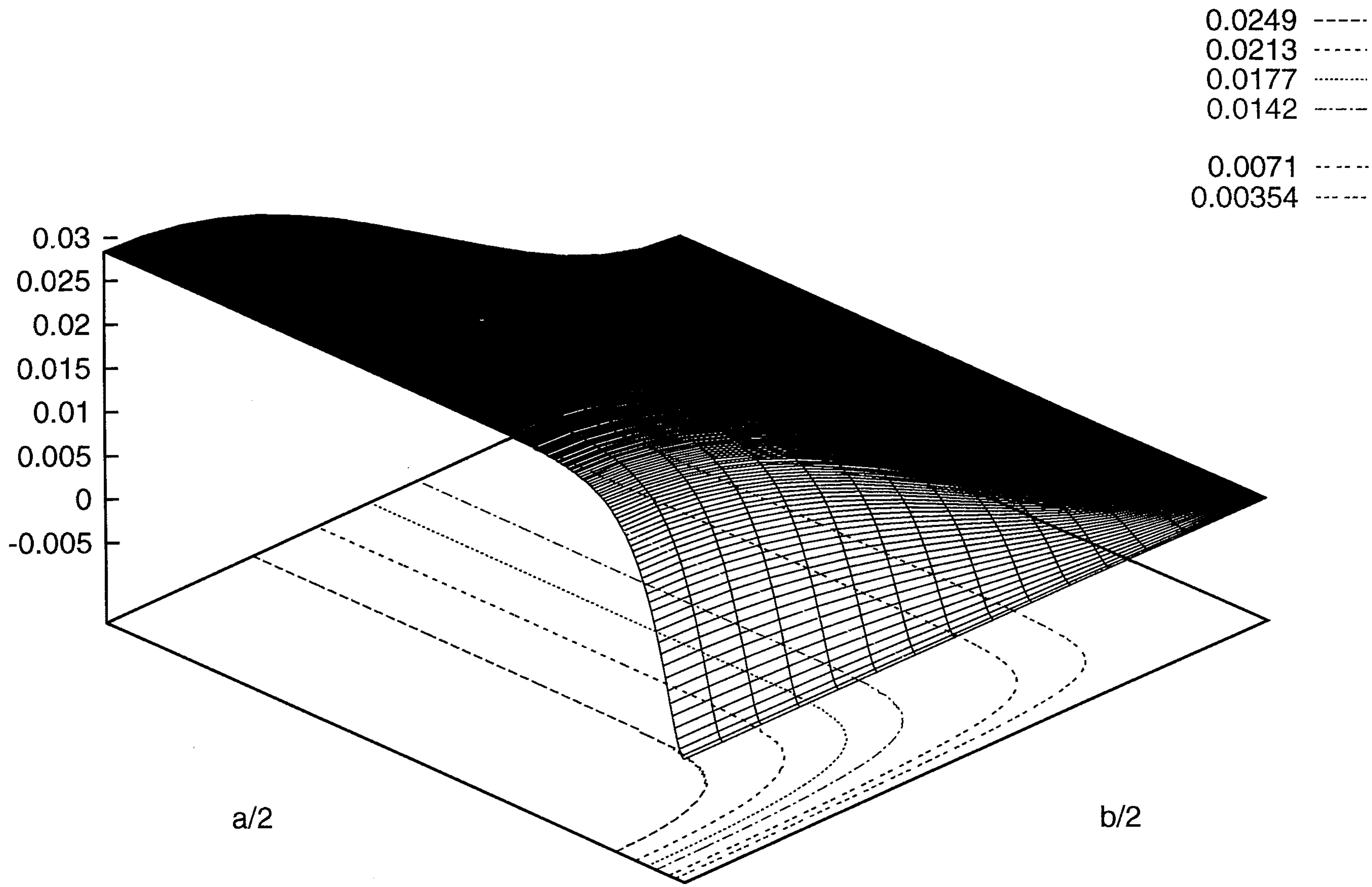


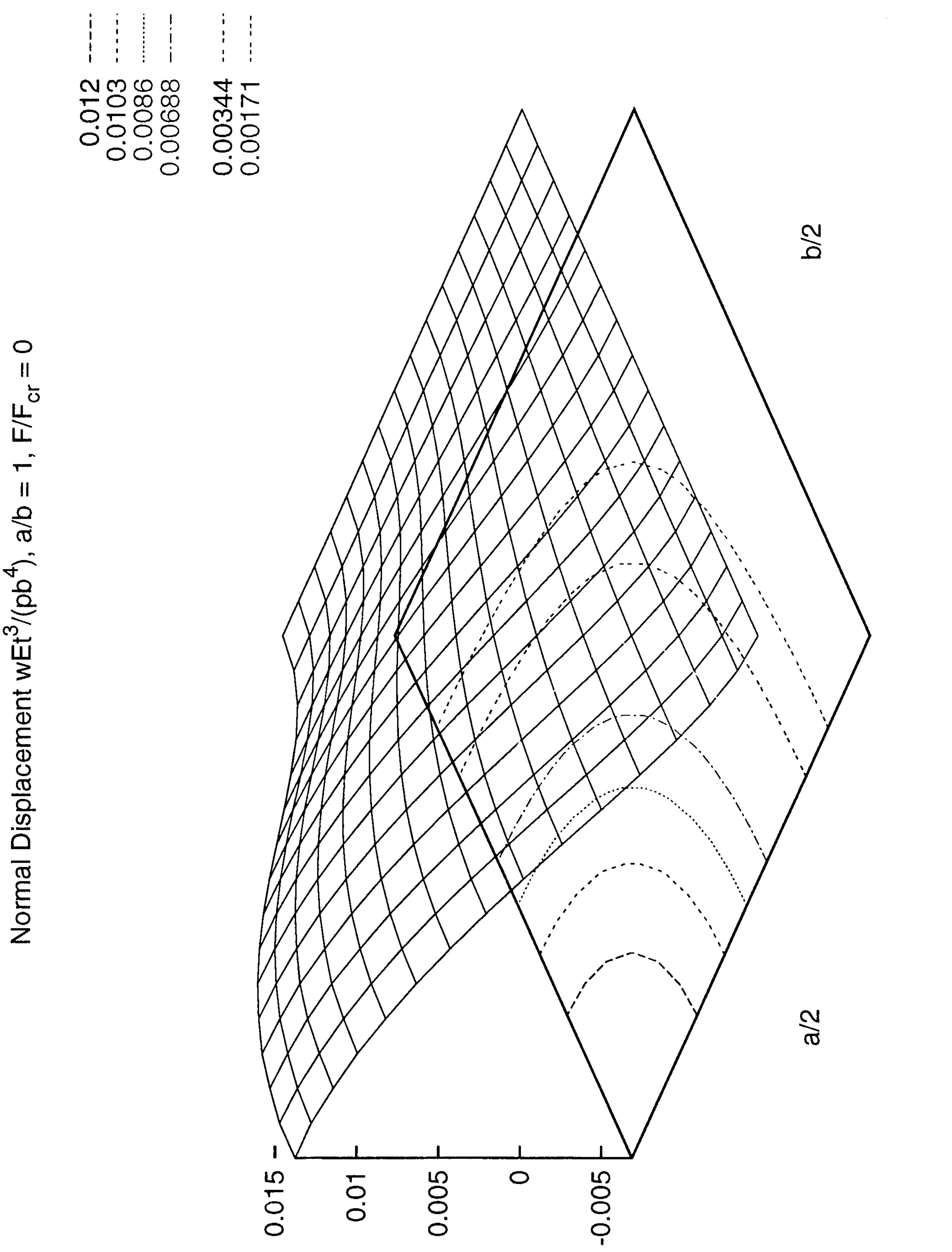
Normal Displacement wEt^3/(pb^4), a/b = 2, F/F_cr = 1.5
0.0194
0.0166
0.0139
0.0111
0.00554
0.00277
0.025
0.02
0.015
0.01
0.005
0
-0.005
a/2
b/2

Normal Displacement $wEt^3/(pb^4)$, $a/b = 4$, $F/F_{cr} = 1.5$



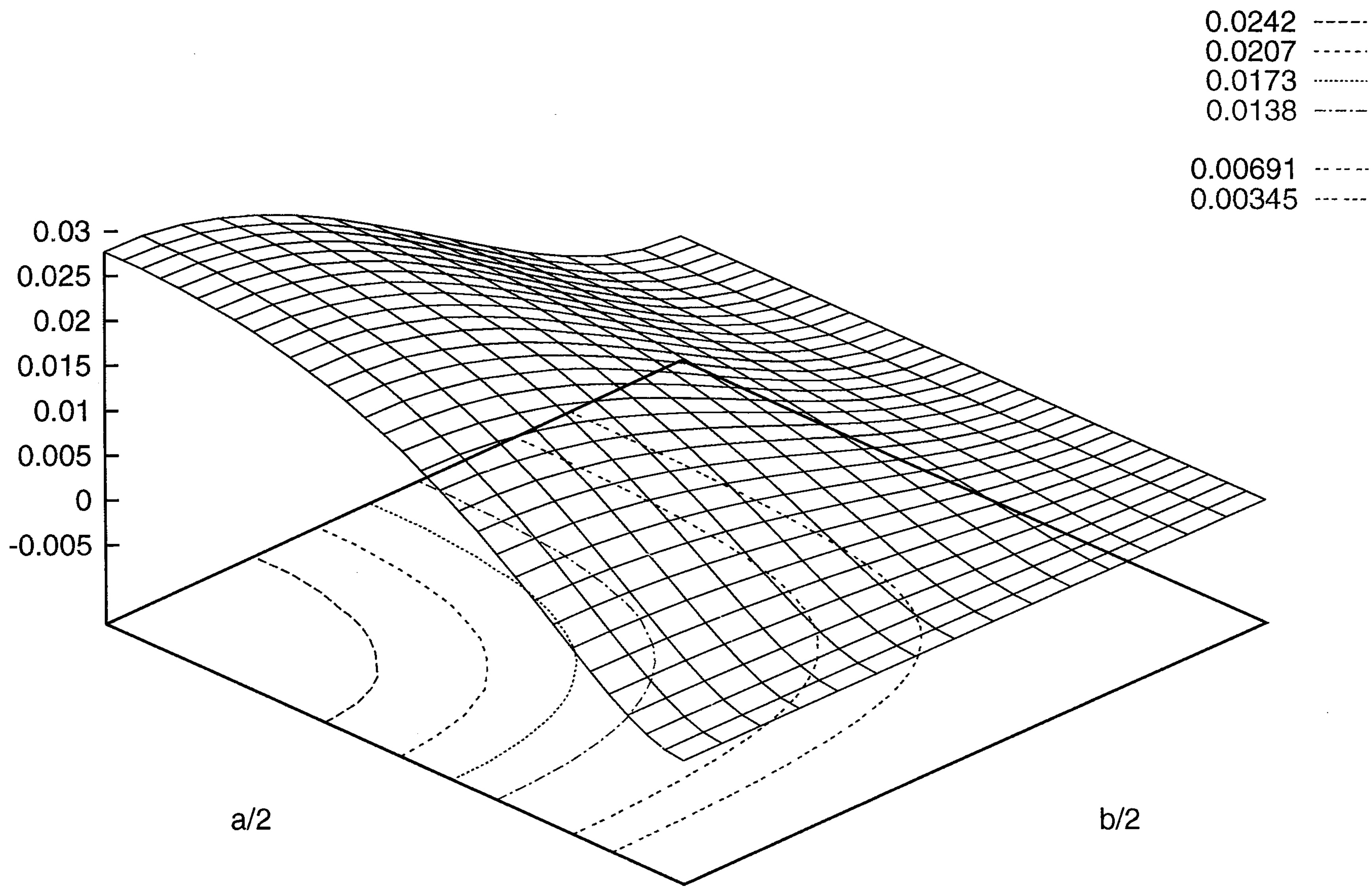
Normal Displacement $wEt^3/(pb^4)$, $a/b = 16$, $F/F_{cr} = 1.5$



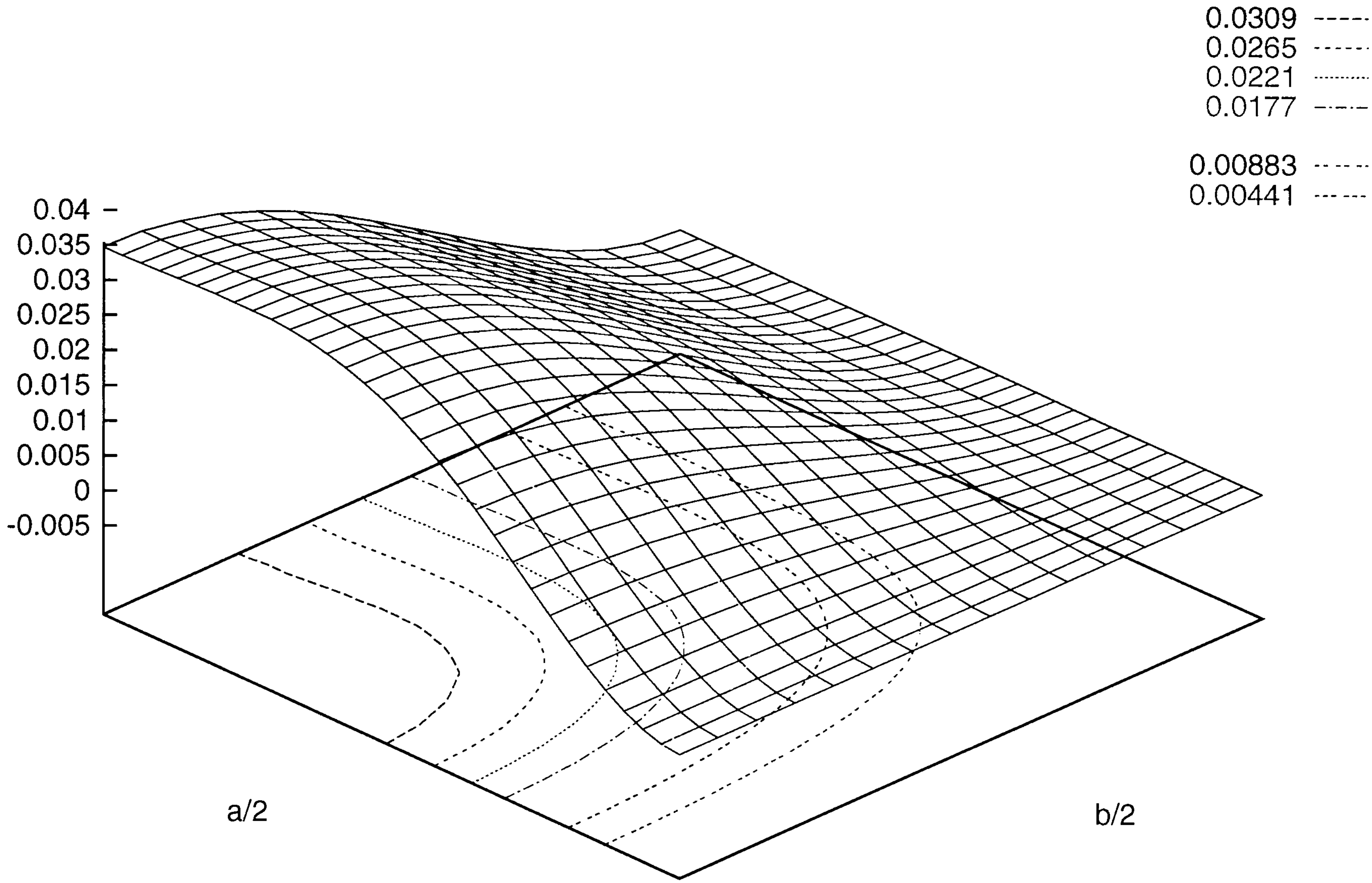


Normal Displacement wEt^3/(pb^4), a/b = 1, F/F_cr = 0
0.012
0.0103
0.0086
0.00688
0.00344
0.00171
0.015
0.01
0.005
0
-0.005
a/2
b/2

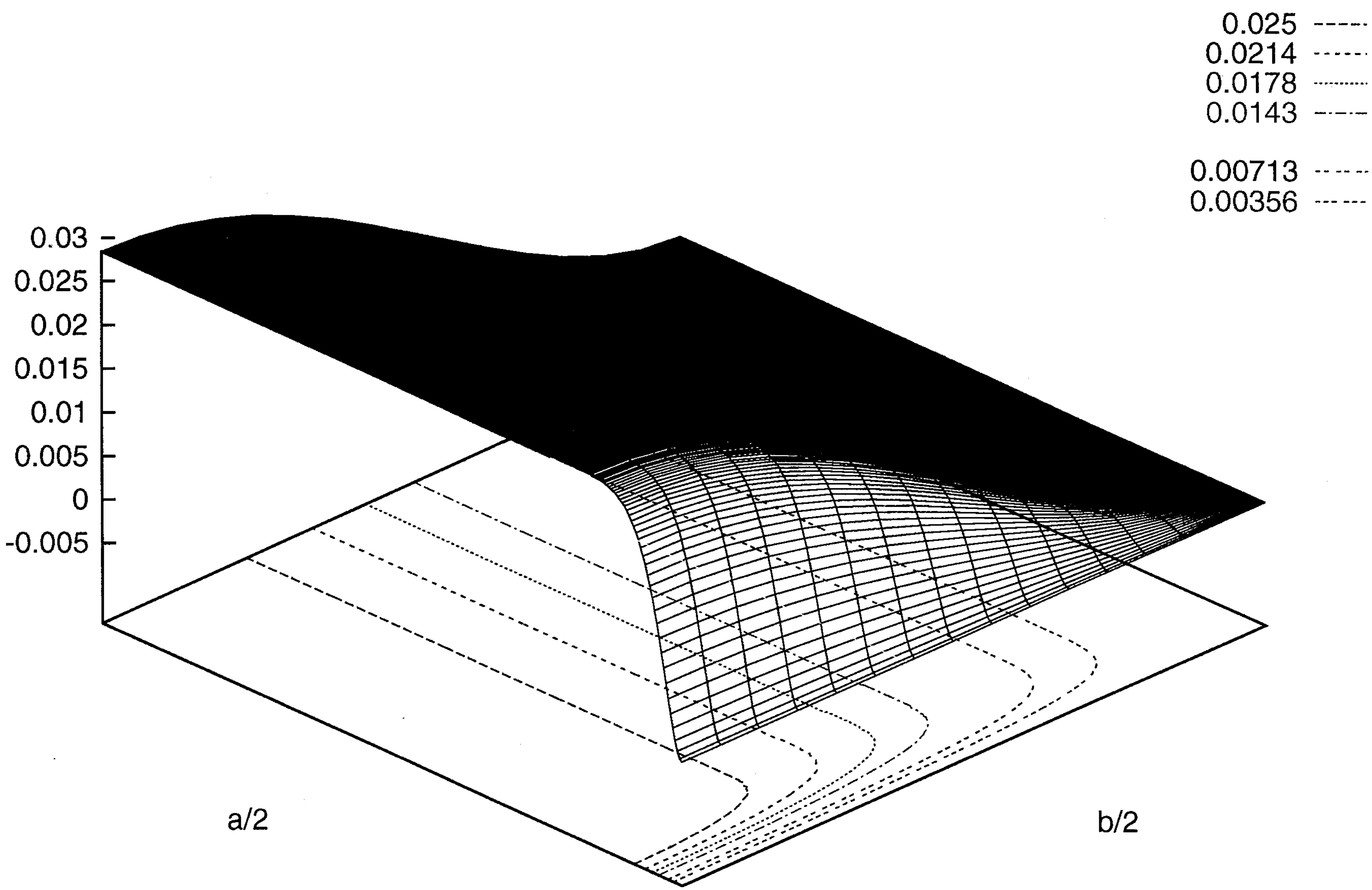
Normal Displacement $wEt^3/(pb^4)$, $a/b = 2$, $F/F_{cr} = 0$



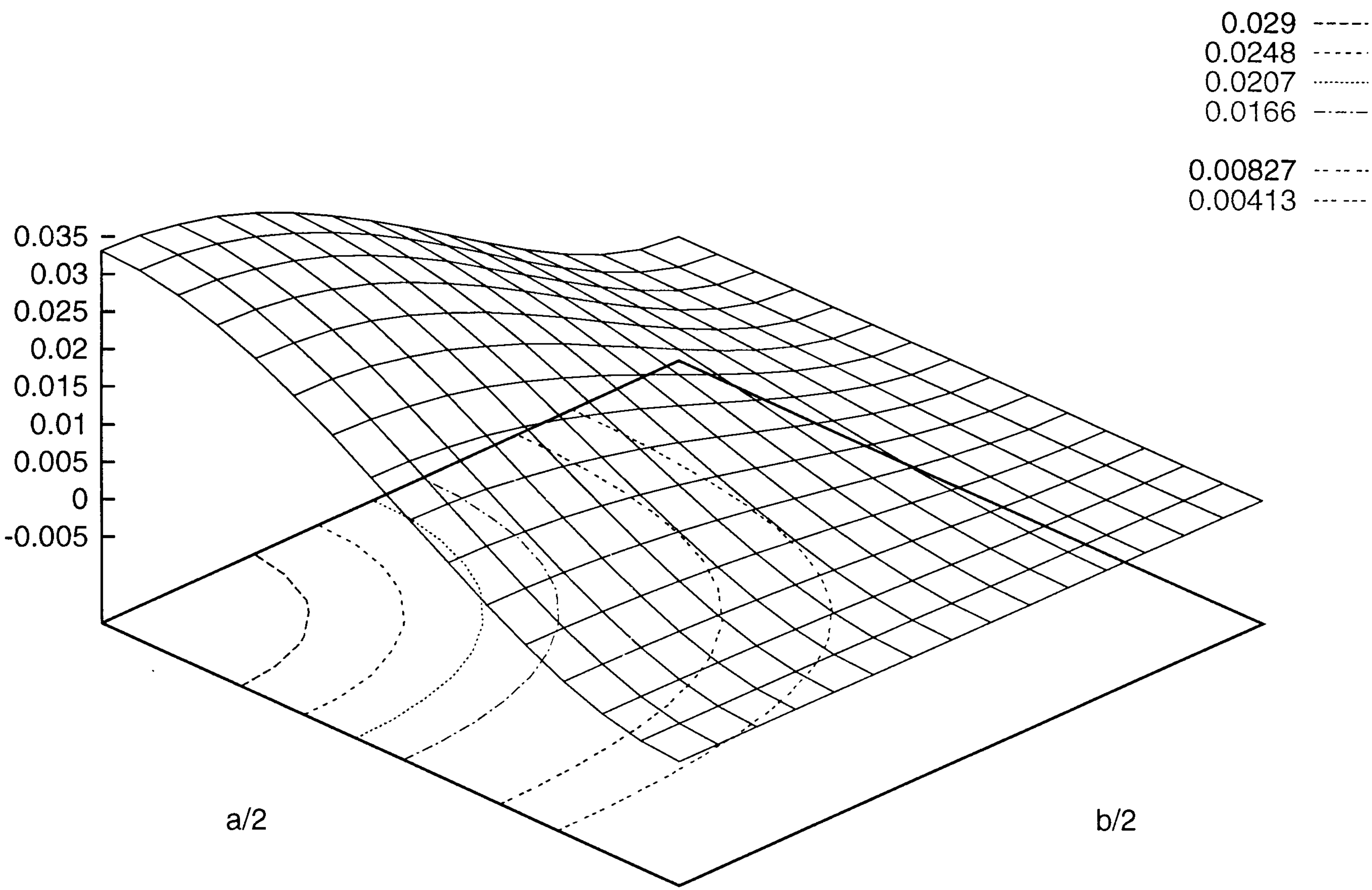
Normal Displacement $wEt^3/(pb^4)$, $a/b = 4$, $F/F_{cr} = 0$



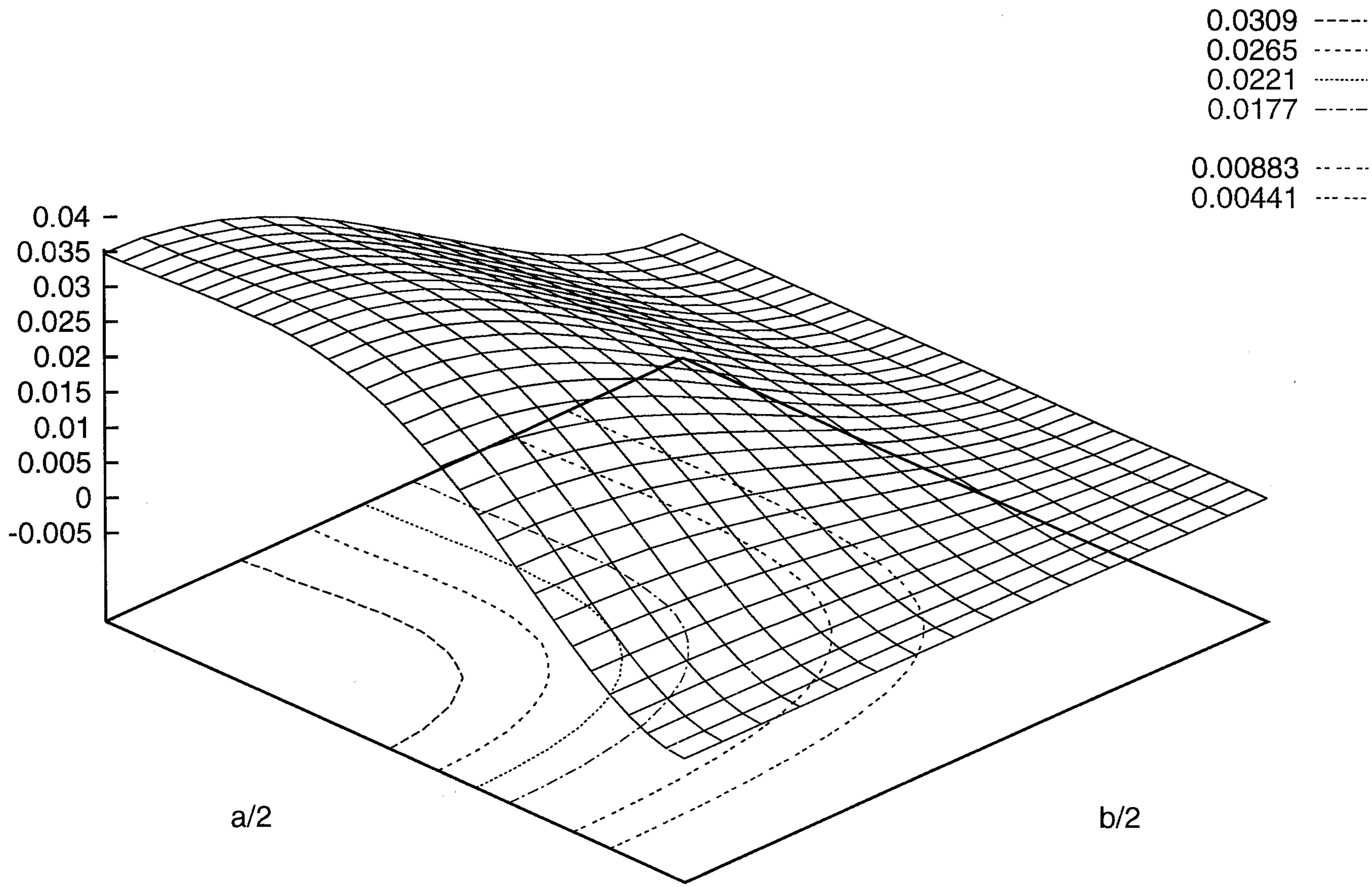
Normal Displacement $wEt^3/(pb^4)$, $a/b = 16$, $F/F_{cr} = 0$



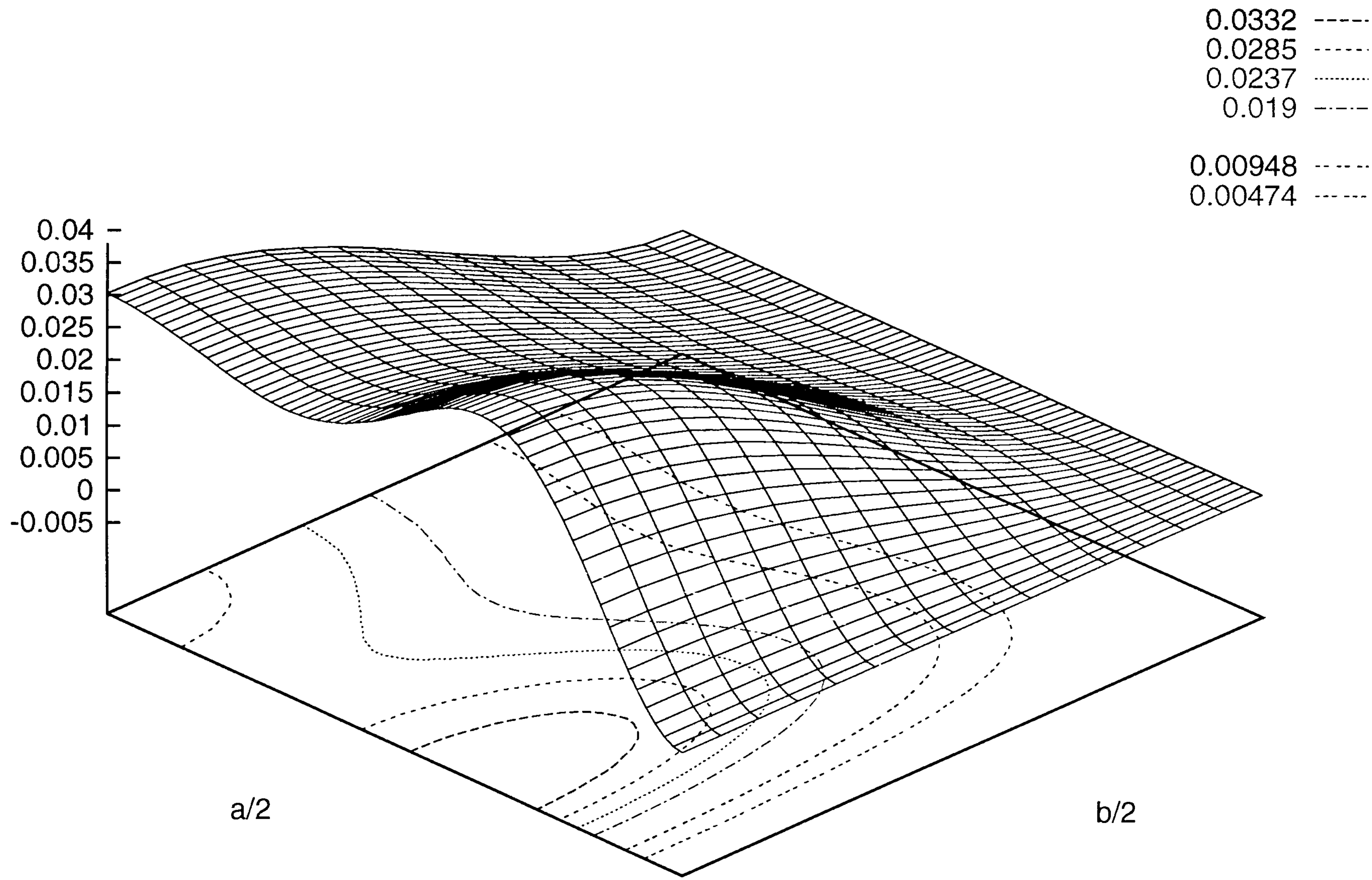
Normal Displacement $wEt^3/(pb^4)$, $a/b = 1$, $F/F_{cr} = -1.5$



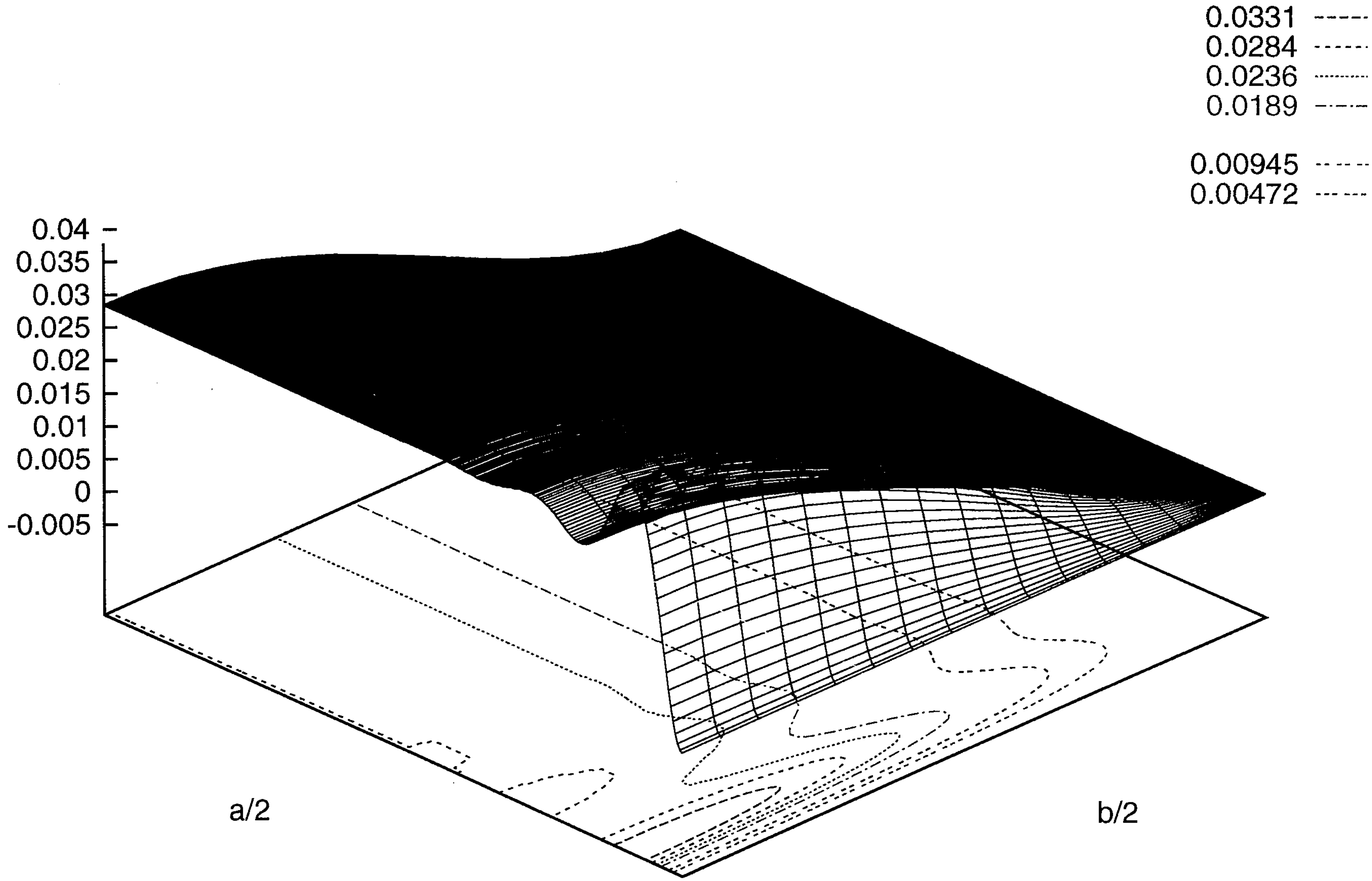
Normal Displacement $wEt^3/(pb^4)$, $a/b = 2$, $F/F_{cr} = -1.5$

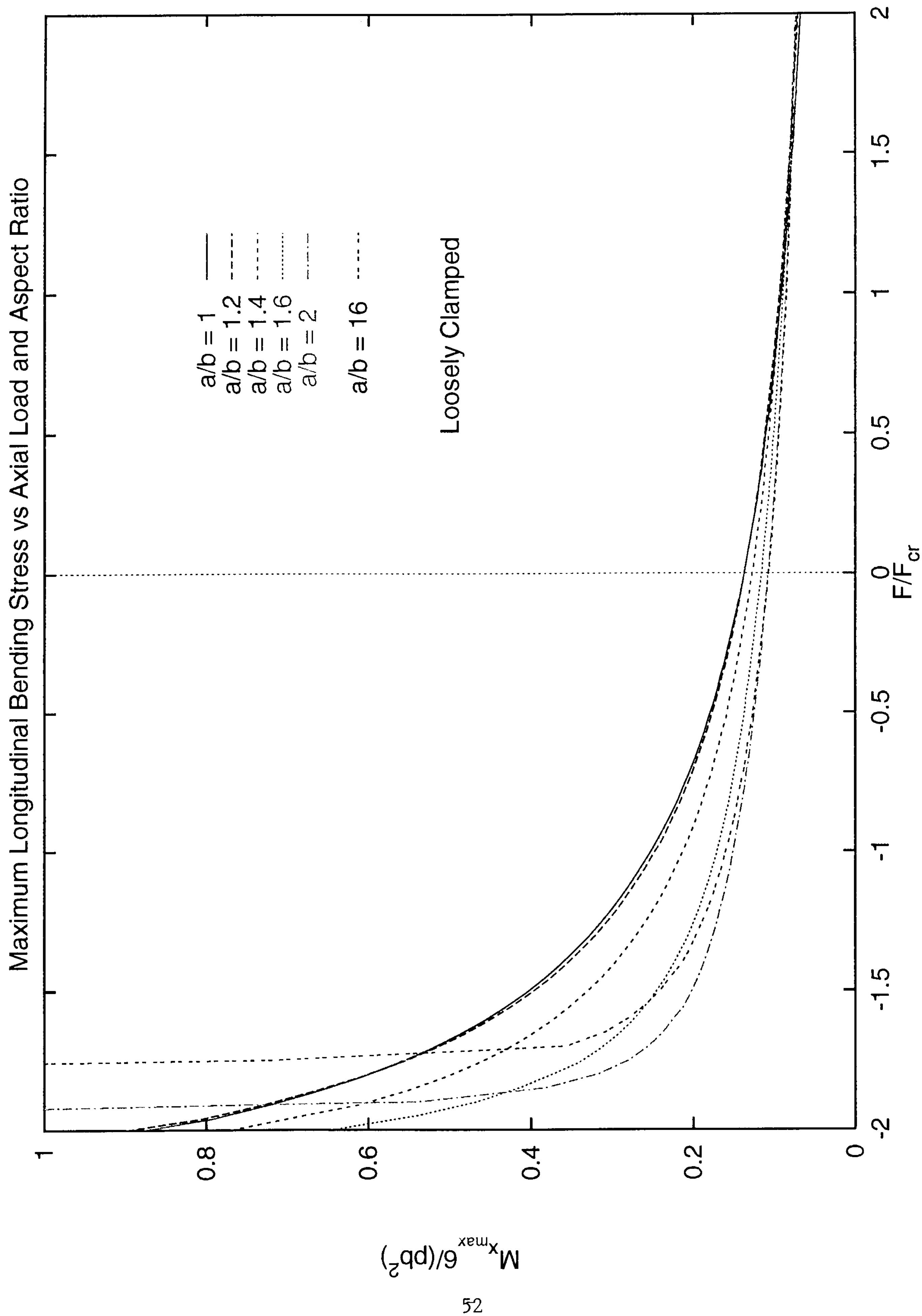


Normal Displacement $wEt^3/(pb^4)$, $a/b = 4$, $F/F_{cr} = -1.5$

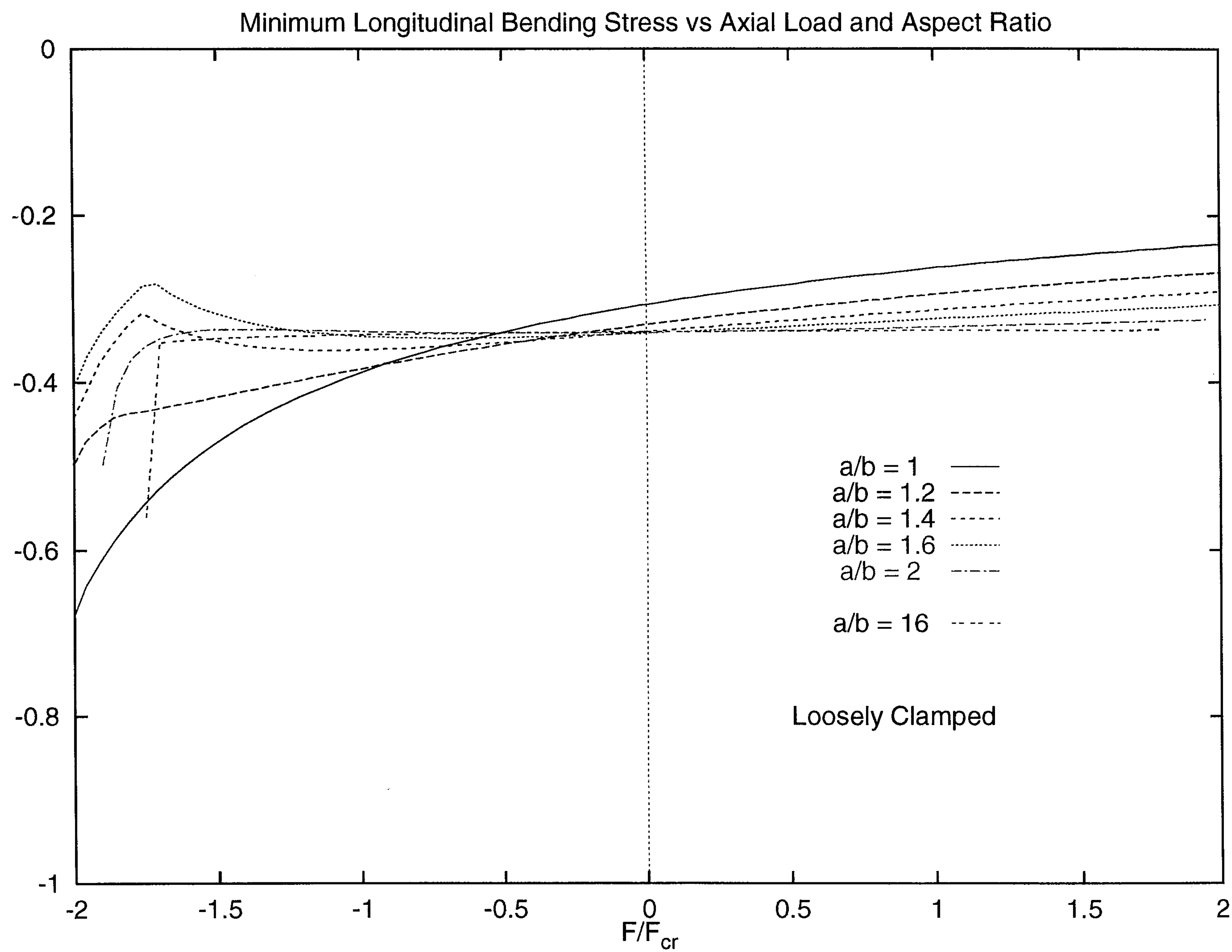


Normal Displacement $wEt^3/(pb^4)$, $a/b = 16$, $F/F_{cr} = -1.5$

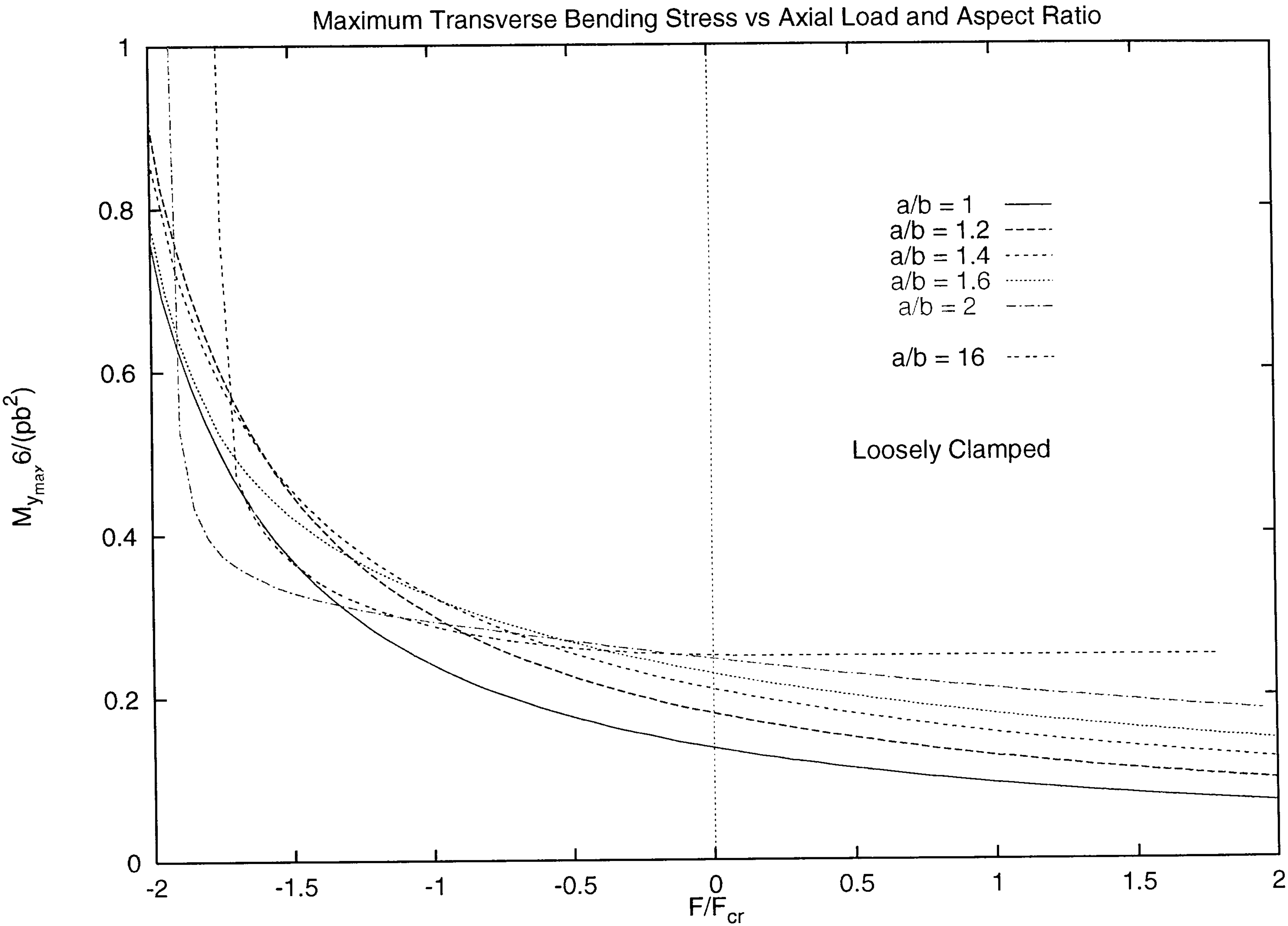




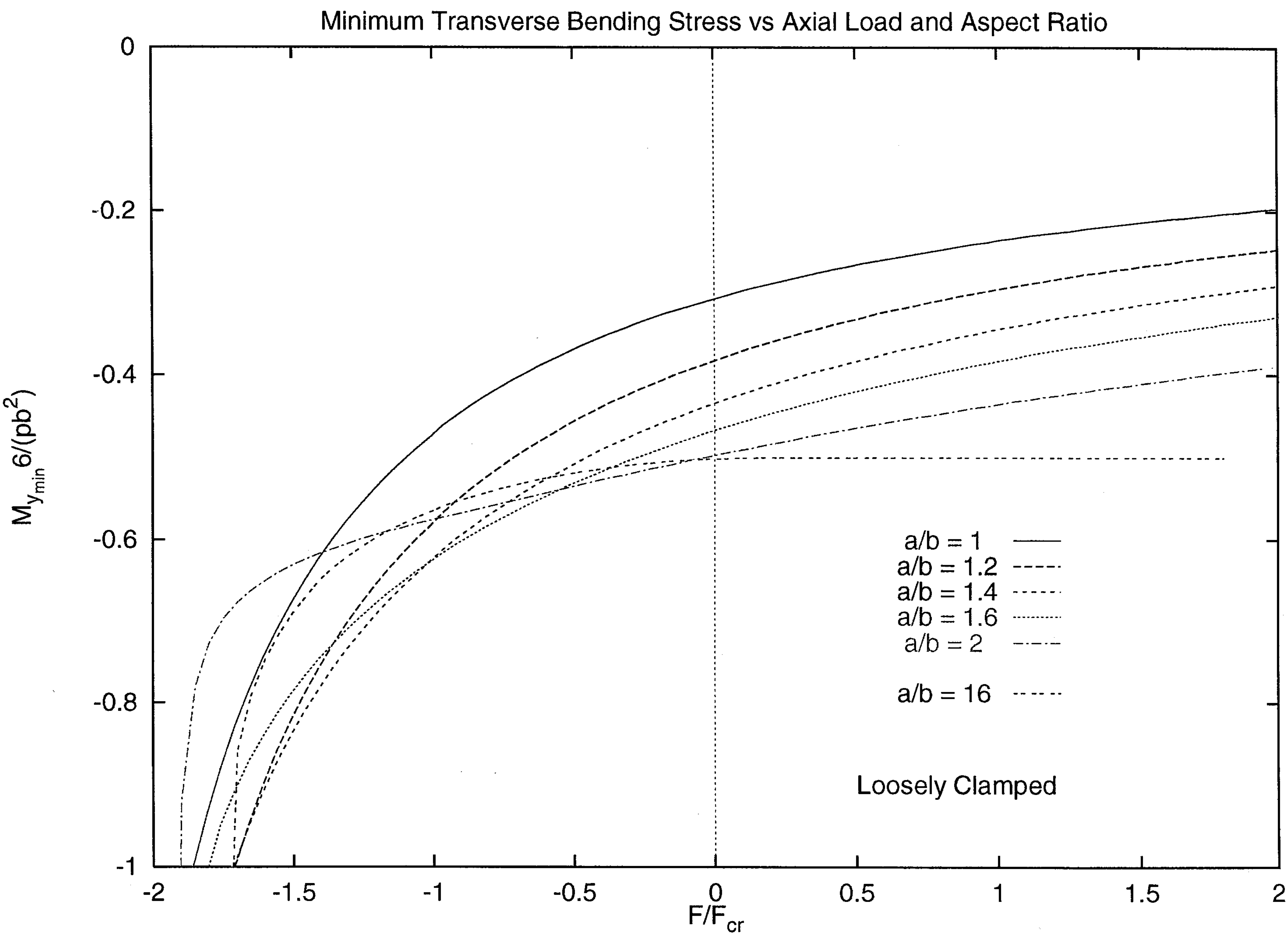
Maximum Longitudinal Bending Stress vs Axial Load and Aspect Ratio
a/b = 1
a/b = 1.2
a/b = 1.4
a/b = 1.6
a/b = 2
a/b = 16
Loosely Clamped
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
F/F_cr
0
0.2
0.4
0.6
0.8
1
M_x_max 6/(pb^2)

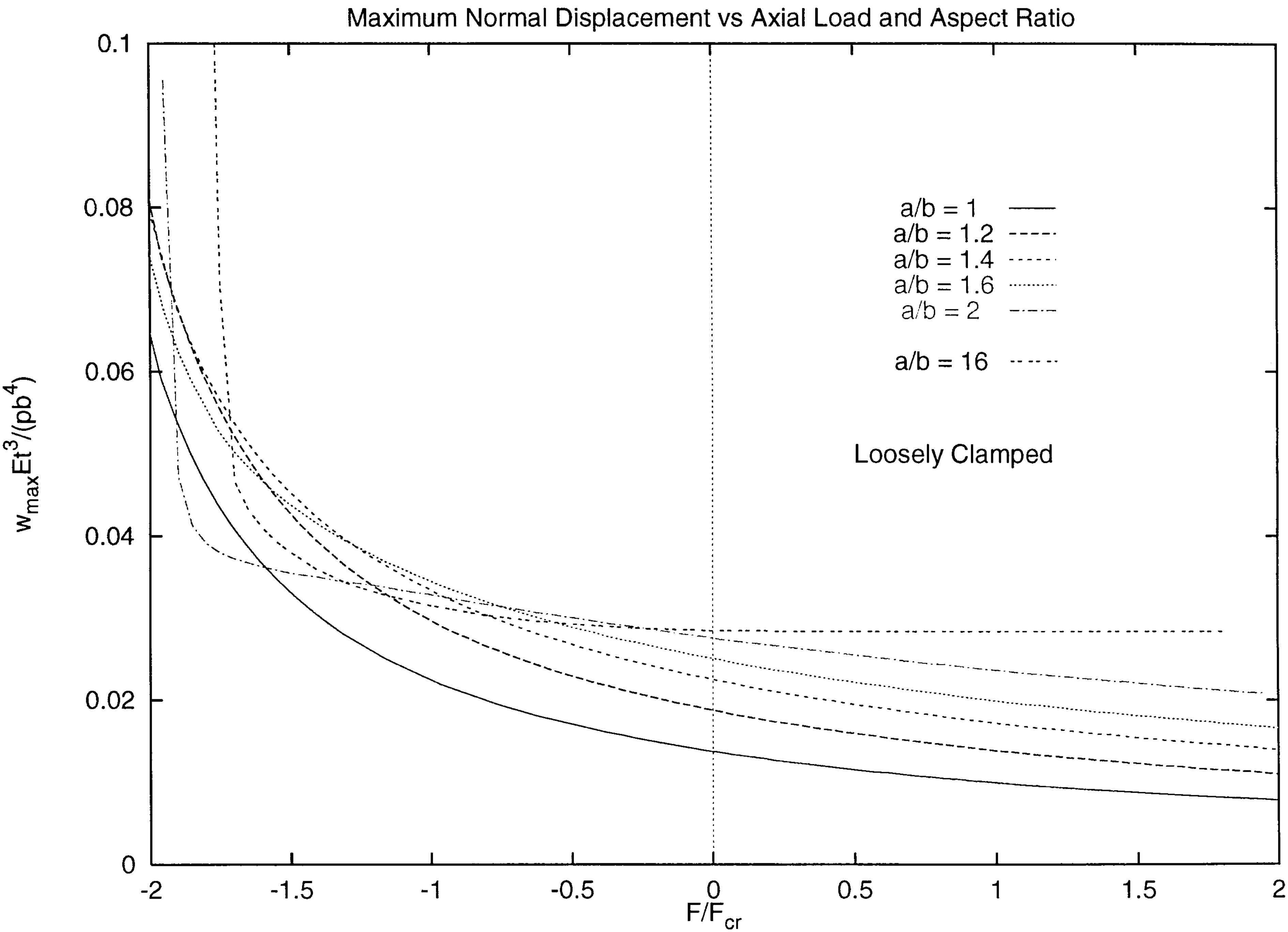


Minimum Longitudinal Bending Stress vs Axial Load and Aspect Ratio
0
-0.2
-0.4
-0.6
-0.8
-1
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
$M_{x_{min}} 6/(pb^2)$
F/F_{cr}
a/b = 1
a/b = 1.2
a/b = 1.4
a/b = 1.6
a/b = 2
a/b = 16
Loosely Clamped



Maximum Transverse Bending Stress vs Axial Load and Aspect Ratio
a/b = 1
a/b = 1.2
a/b = 1.4
a/b = 1.6
a/b = 2
a/b = 16
Loosely Clamped
$M_{y_{max}} 6/(pb^2)$
F/F_{cr}
0
0.2
0.4
0.6
0.8
1
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2


Minimum Transverse Bending Stress vs Axial Load and Aspect Ratio
0
-0.2
-0.4
-0.6
-0.8
-1
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
F/F_cr
M_y_min 6/(pb^2)
a/b = 1
a/b = 1.2
a/b = 1.4
a/b = 1.6
a/b = 2
a/b = 16
Loosely Clamped


Maximum Normal Displacement vs Axial Load and Aspect Ratio
a/b = 1
a/b = 1.2
a/b = 1.4
a/b = 1.6
a/b = 2
a/b = 16
Loosely Clamped
$w_{max}Et^3/(pb^4)$
F/F_{cr}
0.1
0.08
0.06
0.04
0.02
0
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2

Additional Membrane Stresses

We next studied the importance of nonlinearities by solving equation (9) with no imperfection and the clamped boundary conditions (4) or (5). The graphs show the nondimensionalized maximum and minimum additional membrane stresses due to bending. Note that the additional membrane stresses are at least an order of magnitude smaller than those in the preceding section from the linear equations. Note also that the main effect of preventing lateral displacement of the edges $y = \pm\frac{b}{2}$ is to increase the additional transverse membrane stress, whereas the additional longitudinal membrane stress is unaffected.

For these graphs the following parameter values which are typical of ship plating were chosen:

$E = 30,000,000$ psi

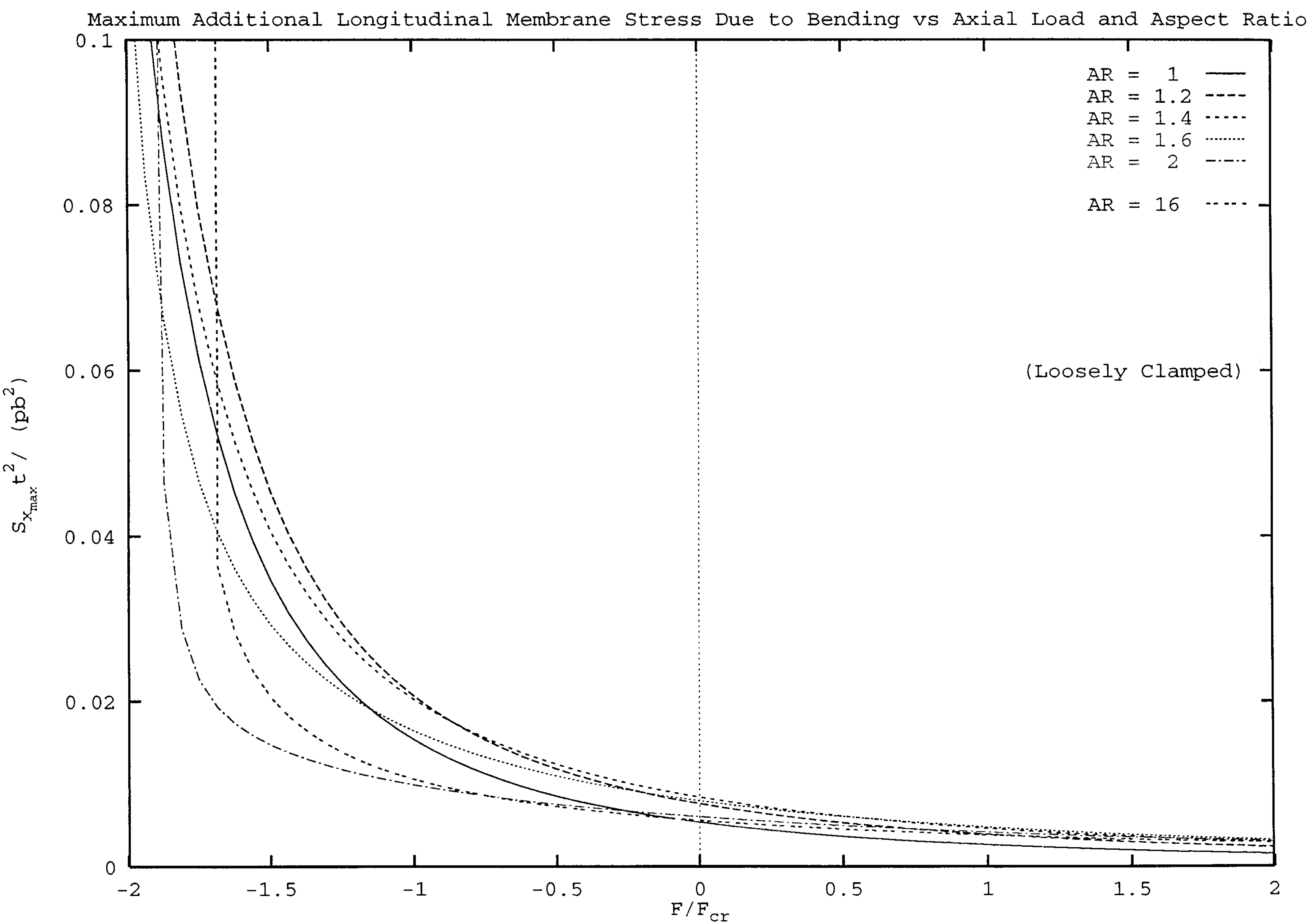
$\nu = .3$

$b = 36$ in

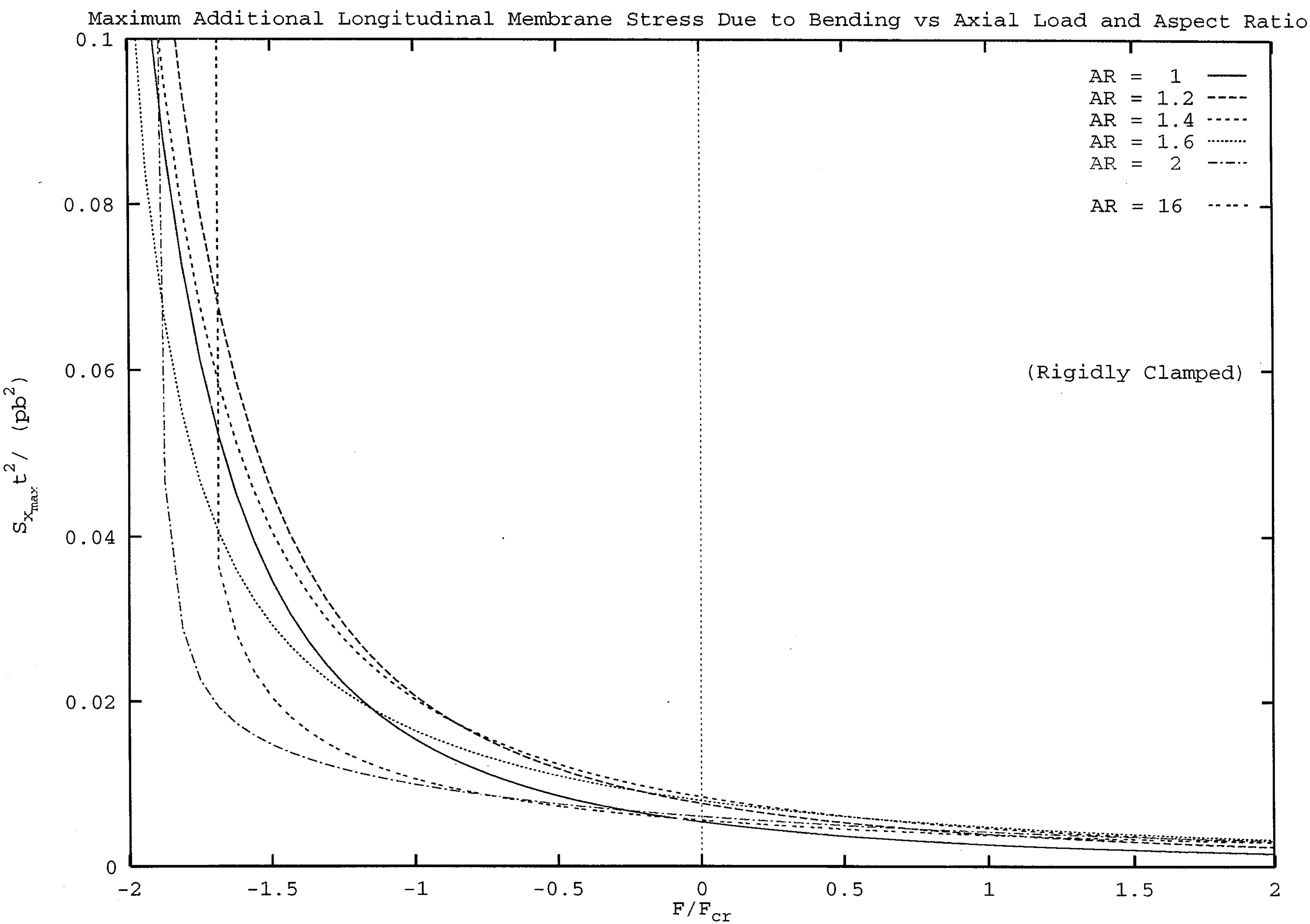
$t = .5$ in

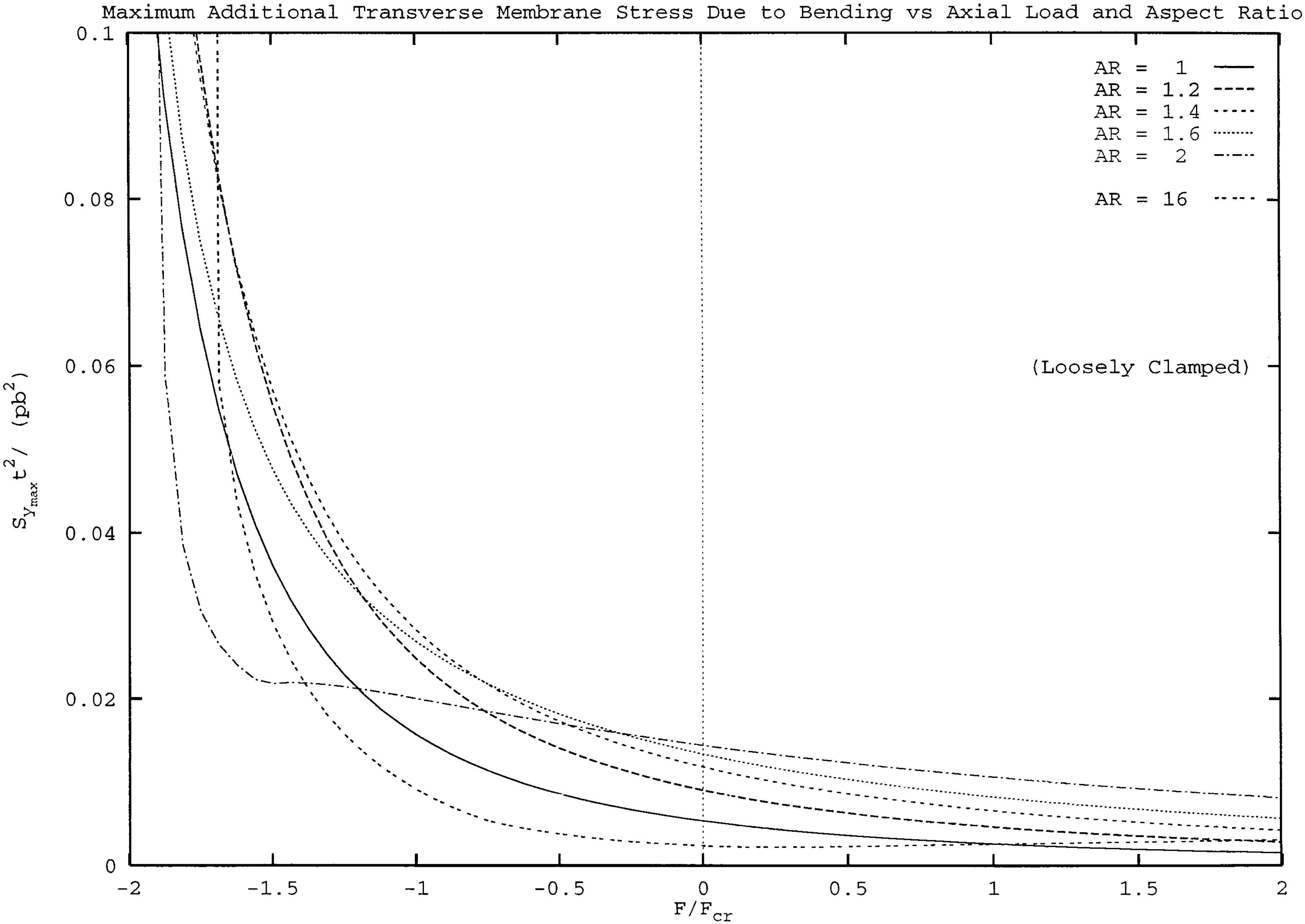
$p = 18$ psi

The additional membrane stresses corresponding to other parameter values may be obtained from these same graphs with the vertical axis scaled in accordance with (10).



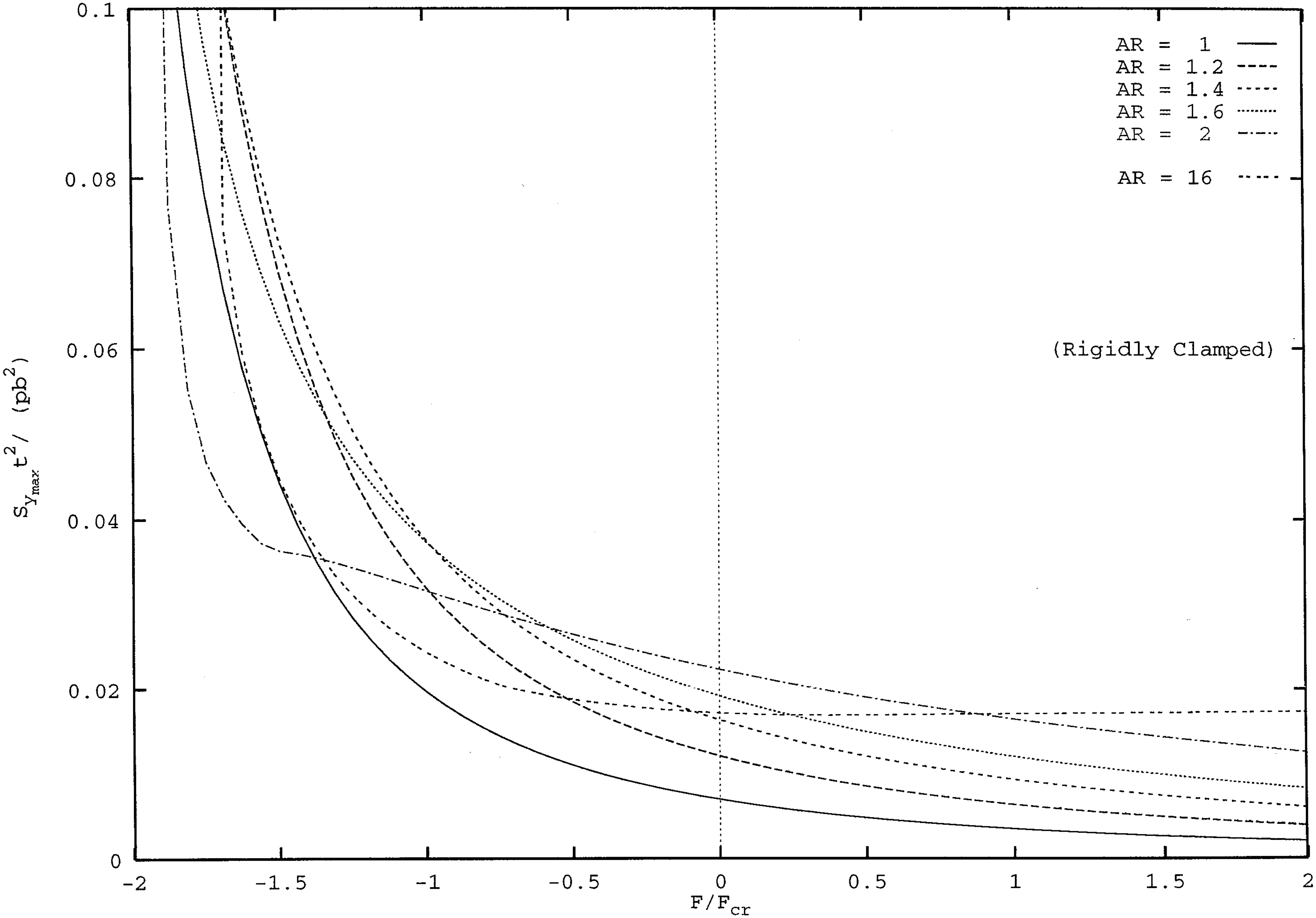
Maximum Additional Longitudinal Membrane Stress Due to Bending vs Axial Load and Aspect Ratio
AR = 1
AR = 1.2
AR = 1.4
AR = 1.6
AR = 2
AR = 16
(Loosely Clamped)
$S_{x_{max}} t^2 / (pb^2)$
F/F_{cr}
0.1
0.08
0.06
0.04
0.02
0
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2


Maximum Additional Longitudinal Membrane Stress Due to Bending vs Axial Load and Aspect Ratio
AR = 1
AR = 1.2
AR = 1.4
AR = 1.6
AR = 2
AR = 16
(Rigidly Clamped)
$S_{x_{max}} t^2 / (pb^2)$
F/F_{cr}
0.1
0.08
0.06
0.04
0.02
0
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2

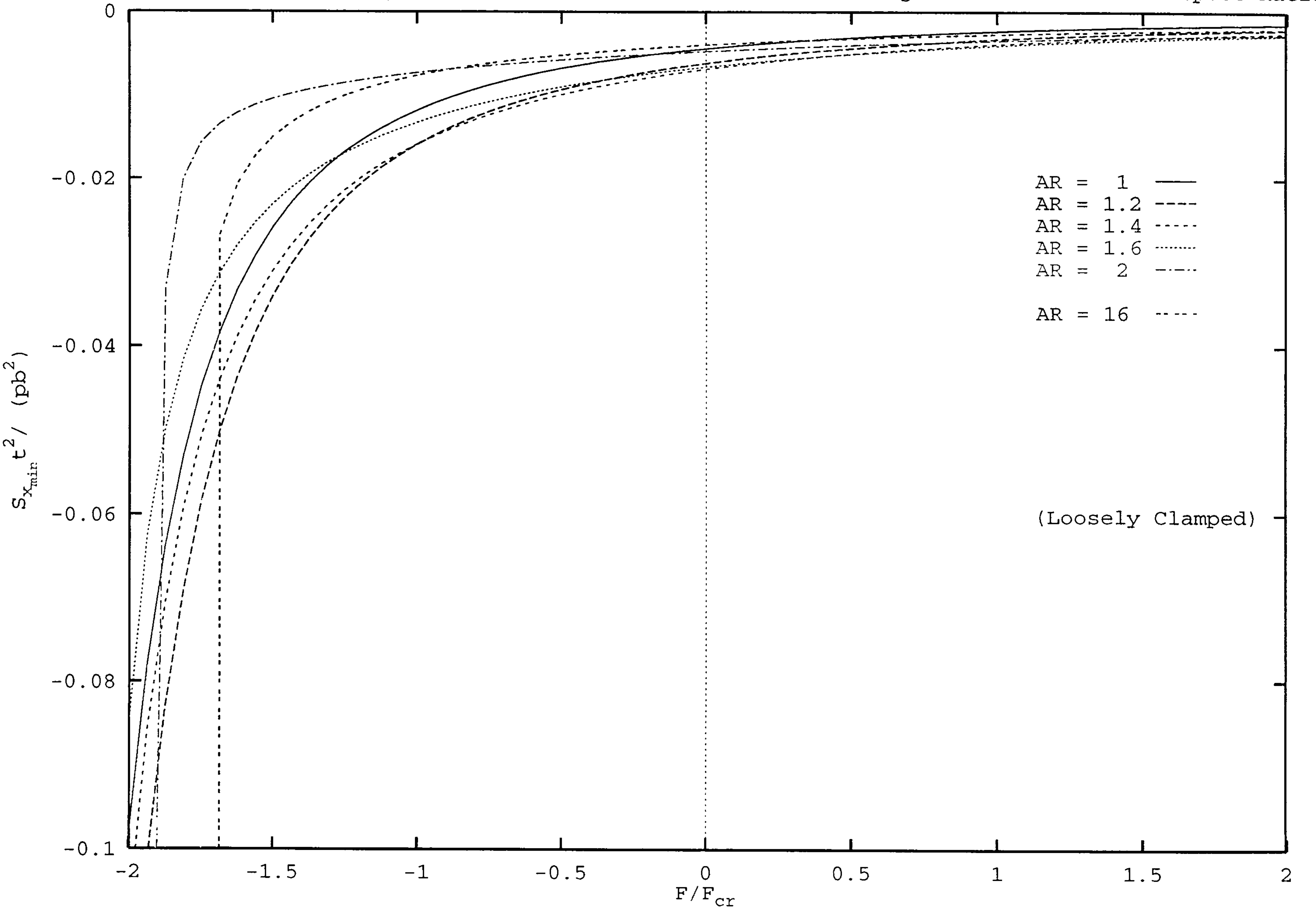


Maximum Additional Transverse Membrane Stress Due to Bending vs Axial Load and Aspect Ratio
AR = 1
AR = 1.2
AR = 1.4
AR = 1.6
AR = 2
AR = 16
(Loosely Clamped)
0.1
0.08
0.06
0.04
0.02
0
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
F/F_cr
S_ymax t^2/ (pb^2)

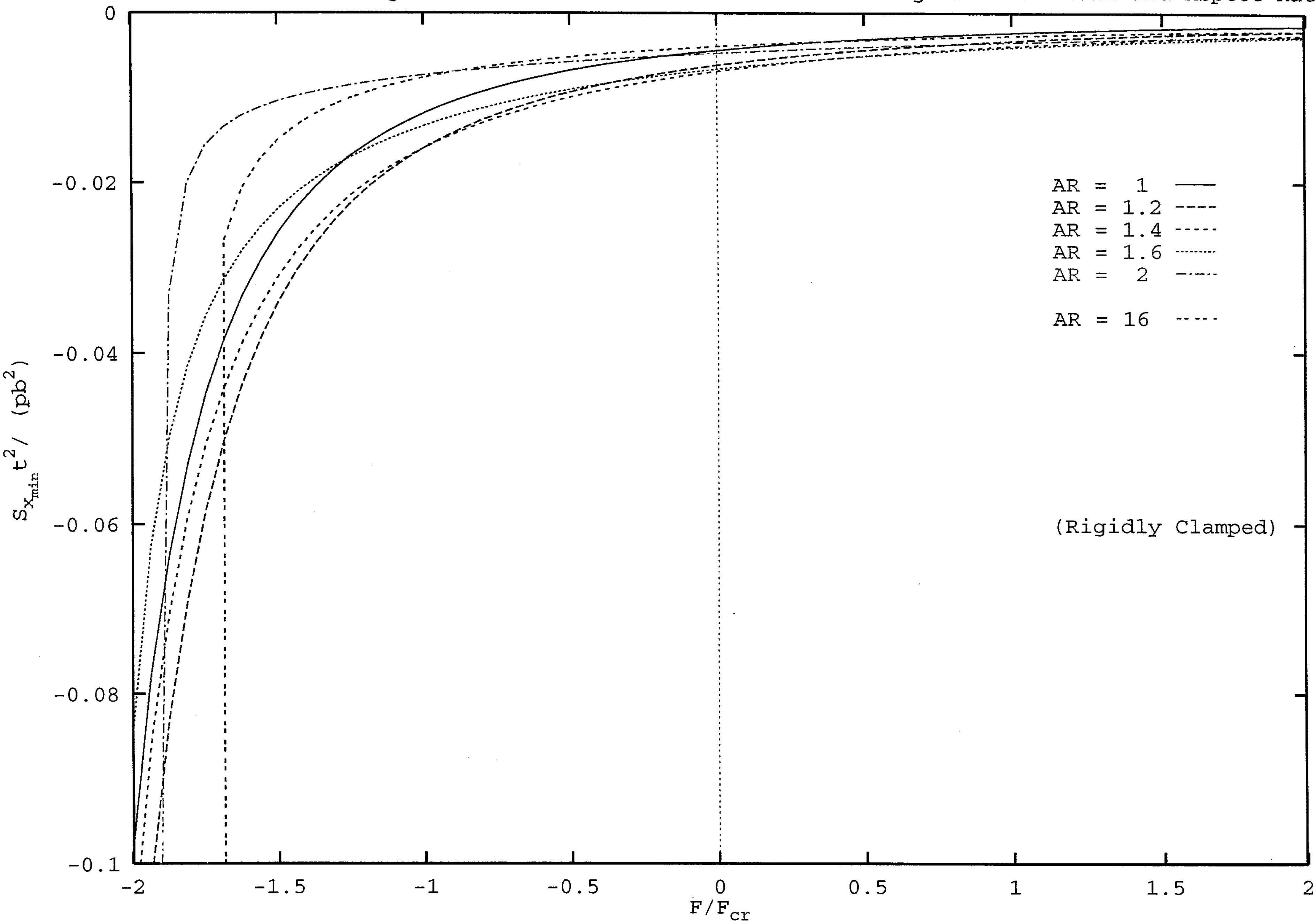
Maximum Additional Transverse Membrane Stress Due to Bending vs Axial Load and Aspect Ratio

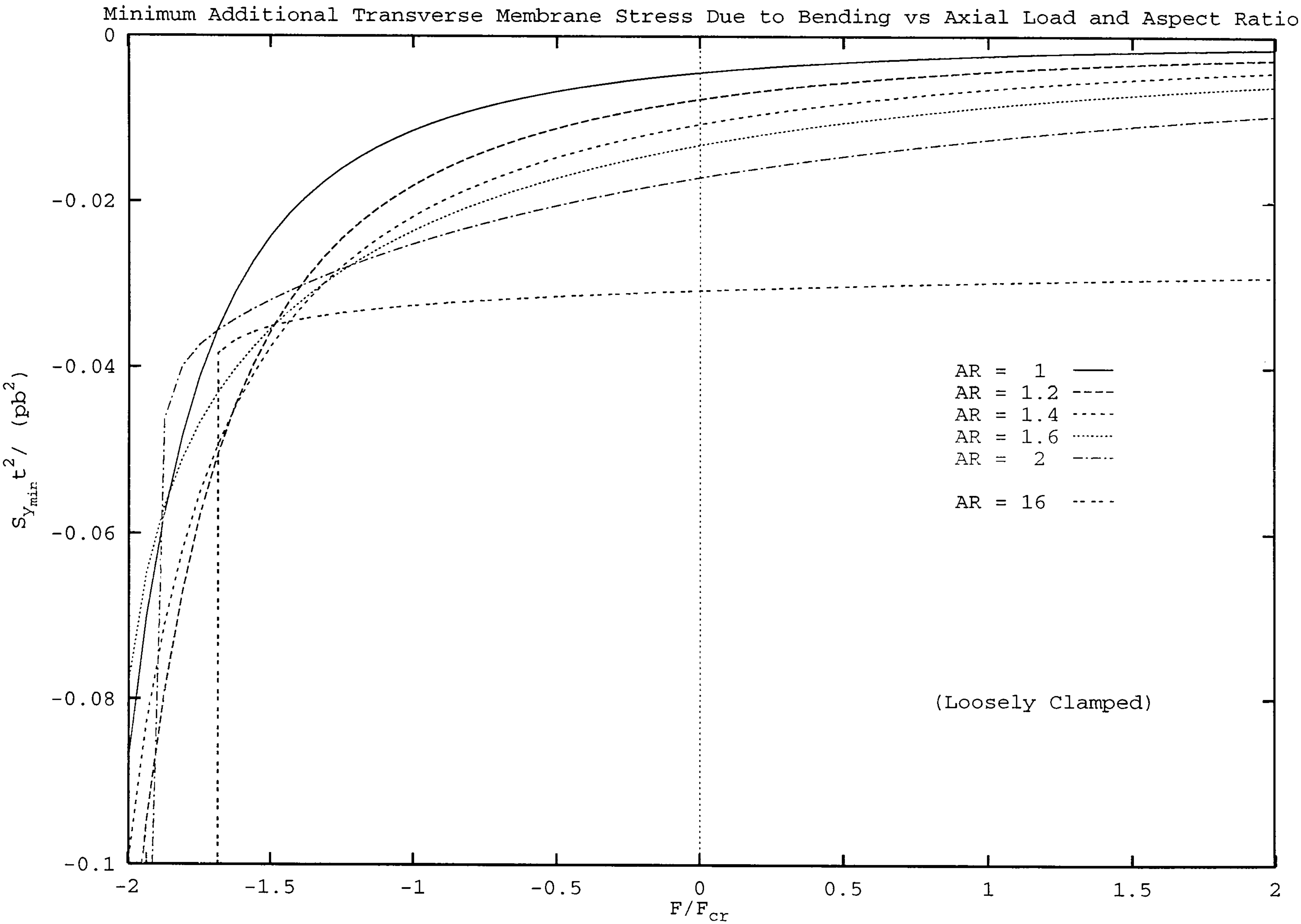


Minimum Additional Longitudinal Membrane Stress Due to Bending vs Axial Load and Aspect Ratio



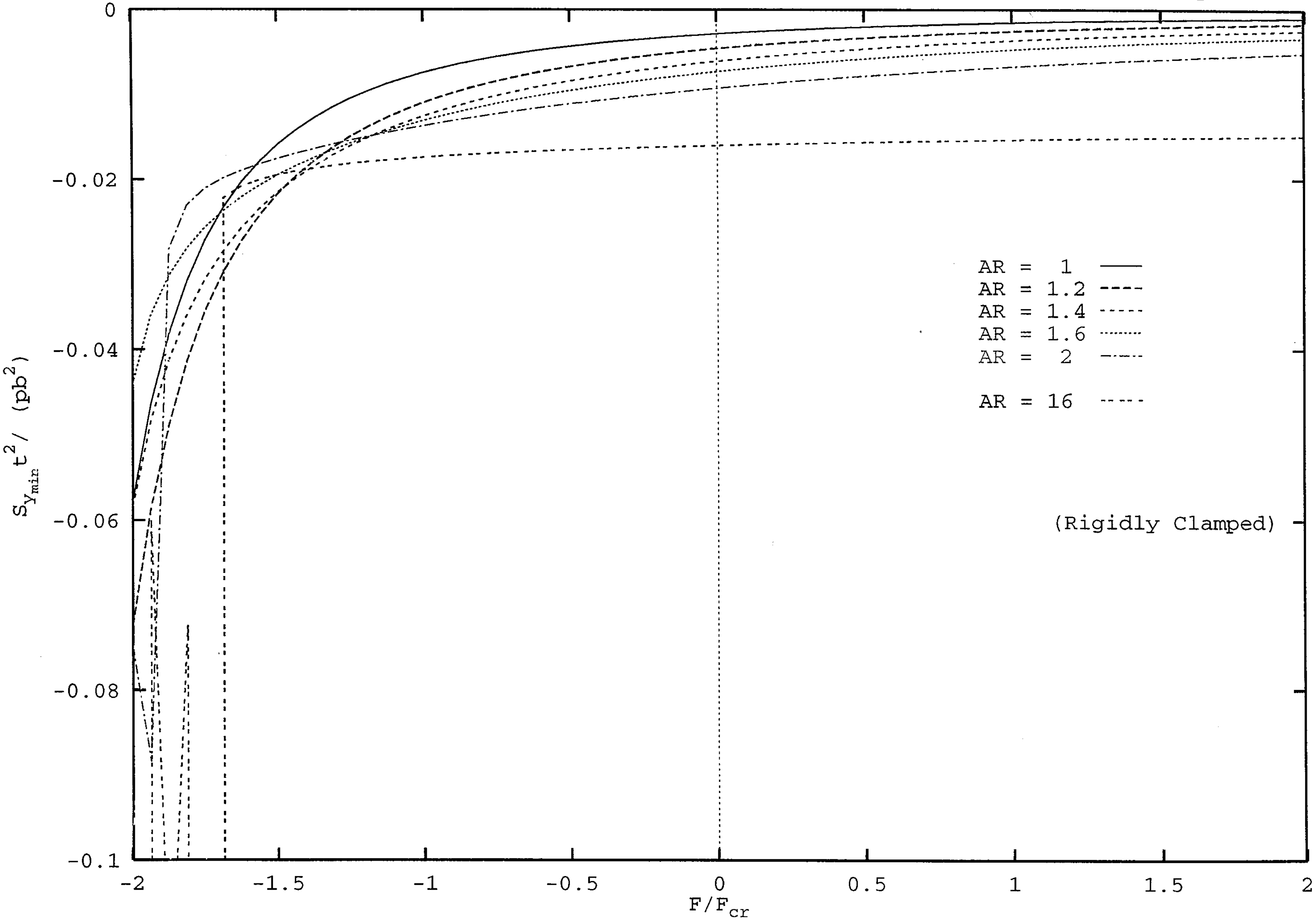
Minimum Additional Longitudinal Membrane Stress Due to Bending vs Axial Load and Aspect Ratio





Minimum Additional Transverse Membrane Stress Due to Bending vs Axial Load and Aspect Ratio
0
-0.02
-0.04
-0.06
-0.08
-0.1
$S_{y_{min}} t^2 / (pb^2)$
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
F/F_{cr}
AR = 1
AR = 1.2
AR = 1.4
AR = 1.6
AR = 2
AR = 16
(Loosely Clamped)

Minimum Additional Transverse Membrane Stress Due to Bending vs Axial Load and Aspect Ratio



Imperfections

Finally, we solved the linearized equations (7)-(8) with no pressure, an imperfection, and the simply-supported boundary conditions (6). Let us consider imperfection shapes of the form

$$w_0 = \cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b} \text{ with } m = 1,3,5,\ldots \text{ and } n = 1,3,5,\ldots$$

or

$$w_0 = \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \text{ with } m = 2,4,6,\ldots \text{ and } n = 2,4,6,\ldots$$

where m and n are the number of half waves in the longitudinal and transverse directions, respectively. For these shapes we obtain the analytical solution

$$w = -\frac{w_0}{\frac{F_{cr}}{F}\left(\frac{mb}{2a}+\frac{n^2a}{2mb}\right)^2+1}$$

The bending stress resultants then follow from (3) by differentiation:

$$M_x = -\frac{\pi^2 D\left(\frac{m^2}{a^2}+\frac{\nu n^2}{b^2}\right)w_0}{\frac{F_{cr}}{F}\left(\frac{mb}{2a}+\frac{n^2a}{2mb}\right)^2+1}, \quad \text{etc.}$$

Of course, to prevent buckling we require $\frac{F}{F_{cr}} > -1$.

An alternate imperfection shape proportional to the deflection of the clamped plate under small uniform pressure without inplane load was also allowed in the Fortran code, but for brevity we do not report on this case here.

CONCLUSIONS

We have developed a Fortran code for calculating the displacements and stresses in rectangular plates subjected to the effects of combined loading, various boundary conditions, and imperfections. The next step is to extend the code to a three-dimensional assembly of plates and beams more representative of ship grillages. Progress in the analyses of such structures has recently been made by Danielson *et al* (1988, 1990, 1993, 1994).

ACKNOWLEDGEMENTS

The authors were supported by the Naval Surface Warfare Center (Carderock Division) and by the Naval Postgraduate School Research Program. Dr. D. P. Kihl of the Survivability, Structures, and Materials Directorate at NSWC proposed the title problem and offered several valuable suggestions.

REFERENCES

Bird, M. D. and Steele, C. R. (1991). Separated solution procedure for bending of circular plates with circular holes. *ASME Applied Mechanics Reviews* 44, no. 11, part 2, S27-S35.

Bird, M. D. and Steele, C R. (1992). A solution procedure for Laplace's equation on multiply connected circular domains. *ASME Journal of Applied Mechanics* 114, part 1, 398-404.

Danielson, D. A., and Hodges, D. H. (1988). A beam theory for large global rotation, moderate local rotation, and small strain. *ASME Journal of Applied Mechanics* 55, 179-184.

Danielson, D. A., Kihl, D. P., and Hodges, D. H.(1990). Tripping of thin-walled plating stiffeners in axial compression. *Thin-Walled Structures* 10, 121-142.

Danielson, D. A., Cricelli, A. S., Frenzen, C. L. and Vasudevan, N. (1993). Buckling of stiffened plates under axial compression and lateral pressure. *International Journal of Solids and Structures* 30, 545-551.

Danielson, D. A.,(1994). Analytical tripping loads for stiffened plates. *International Journal of Solids and Structures.* To appear.

Hughes, O. (1983). *Ship Structural Design.* Wiley.

Kang, L. C., Wu, C. H., and Steele, C. R.(1993). Fourier series for polygonal plate bending: A very large plate element. To appear.

Kwok, F. M. W., Kang, L. C., and Steele, C. R.(1991). Mode III fracture mechanics analysis with Fourier series method. *ASME Applied Mechanics Review* 44, 166-170.

Roark, R. J., and Young, W. C.(1975). *Formulas for Stress and Strain.* McGraw-Hill.

Sikora, J. P., Dinsenbacher, A., and Beach, J. E.(1983). A method for estimating lifetime loads and fatigue lives for swath and conventional monohull ships. *Naval Engineers Journal,* 63-85.

Szilard, R. (1974). *Theory and Analysis of Plates.* Prentice-Hall.

Timoshenko, S. P., and Gere, J. M.(1961). *Theory of Elastic Stability.* McGraw-Hill.

Timoshenko, S. P., and Woinowsky-Krieger(1959). *Theory of Plates and Shells.* McGraw-Hill.

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